INDIRECT APPROACH OF DETERMINATION OF COLLECTIVE ALTERNATIVE RANKING ON THE BASIS OF FUZZY EXPERT JUDGEMENTS

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Annotation: *The article suggests methods for determining the collective ranking based on the indirect approach. We consider the case of fuzzy expert preferences given in the form of matrices of fuzzy tournaments and also the case of ordinal fuzzy expert assessments. For aggregation used method for calculating the linguistically quantized speech as well as OWA operator. The first method makes it possible to do without the complex optimization problems that arise in group decision making. Another method can be used for direct ranking of alternatives by experts.*

Keywords: *fuzzy expert judgements, group decision making, group ranking, indirect approach.*

ACM Classification Keywords: *H.4.2 Information Sysyems Applications: Types of Systems: Decision Support.*

Introduction

The methods of group decision making which were called as the collective expert judgement are increasingly frequently used in the applied mathematics and different spheres of human activity. The peculiarity of collective expert judgement as the scientific tool for solving of complex slightly structured problems is fuzziness which is appropriate to the expert judgements.

Setting of the problem

The problem of collective ordinal expert judgment is considered in the following setting [Тоценко, 2006].

Given: finite set of alternatives $A = \{a_1,...,a_{n_A}\}$; qualitative criterion of alternatives judgment (K); normalized coefficients $A = \{a_1,...,a_{n_A}\}$, $l \in N_E = \{1,...,n_E\}$, $\sum_{l=1}^{n_E} \alpha_l = 1$, of expert competence concerning the subject of expertise.

To find: collective alternative ranking of set A according to criterion K, which generalizes the opinion of all experts in the best way and is agreed taking into consideration the expert competence.

Indirect approach provides at least two stages: the stage of expert inormation aggregation and the stage of decision making. *The aggregated (collective, group, social, agreed, etc.) fuzzy* P_C is formed on the aggregation stage. The best alternative or resulting alternative ranking is determined on the stage of decision making on the basis of agreed judgement P_C. It is possible to realize the aggregation stage in different ways according to the kind of individual judgements.

Method of fuzzy collective ordinal judgement determination of alternatives on the basis of fuzzy expert matrixes of paired comparisons

On the aggregation stage of fuzzy expert information from fuzzy individual prefences, given by the experts in the form of paired comparison matrixes, the fuzzy collective preferance P_C in the form of matrix with elements $\mu_{ij}^{(C)}$, $i, j = 1,\dots,n_A$ is built. Each value $\mu_{ij}^{(C)} \in [0,1]$ expresses the level of confidence about preferance of alternative a_i over alternative a_j of expert group in general.

For aggregation of individual preferences we use an approach which is based on the fuzzy majority [Kacprzyk, 1985i, Kacprzyk, 1985ii], according to which we have:

$$\mu_{ij}^{(C)} = \mu_Q\left(\frac{1}{n_E}\sum_{l=1}^{n_E}\mu_{ij}^{(l)}\right) \tag{1}$$

where $\mu_Q(\cdot)$ is membership function of fuzzy quantifier Q, $\mu_{ij}^{(l)}$ is confidence degree about the alternative preference a_i over a_j in the opinion of the l st expert.

Statement 1. Let all individual preferances are fuzzy tournments. If Q is non-decreasing linguistic quantifier with such data as (a,b), that $a+b=1$, then collective preference built according to the rule (1), is also fuzzy tournament.

Proof. Let's choose the arbitrary idexes $i, j \in N_A$.

Let $\frac{1}{n_E}\sum_{l=1}^{n_E}\mu_{ij}^{(l)} \le a$, then $\mu_{ij}^{(C)} = 0$.

$$\begin{gathered}\frac{1}{n_E}\sum_{l=1}^{n_E}\mu_{ij}^{(l)} = \frac{1}{n_E}\sum_{l=1}^{n_E}\left(1-\mu_{ji}^{(l)}\right) = 1-\frac{1}{n_E}\sum_{l=1}^{n_E}\mu_{ji}^{(l)} \le a \Rightarrow \\ \Rightarrow 1-a \le \frac{1}{n_E}\sum_{l=1}^{n_E}\mu_{ji}^{(l)} \Rightarrow b \le \frac{1}{n_E}\sum_{l=1}^{n_E}\mu_{ji}^{(l)} \Rightarrow \mu_{ji}^{(C)} = 1\end{gathered} \tag{2}$$

Suppose now $a < \frac{1}{n_E}\sum_{l=1}^{n_E}\mu_{ij}^{(l)} < b$. Taking into consideration the equivalent transformations in (2), we have the following implications:

$$\begin{gathered}a < 1-\frac{1}{n_E}\sum_{l=1}^{n_E}\mu_{ji}^{(l)} < b \Leftrightarrow a-1 < -\frac{1}{n_E}\sum_{l=1}^{n_E}\mu_{ji}^{(l)} < b-1 \Leftrightarrow \\ \Leftrightarrow 1-b < \frac{1}{n_E}\sum_{l=1}^{n_E}\mu_{ji}^{(l)} < 1-a \Leftrightarrow a < \frac{1}{n_E}\sum_{l=1}^{n_E}\mu_{ji}^{(l)} < b\end{gathered} \tag{3}$$

Further, taking into consideratino (3), we have:

$$\mu_{ij}^{(Q)} + \mu_{ji}^{(Q)} = \frac{\frac{1}{n_E}\sum_{l=1}^{n_E}\mu_{ij}^{(l)} - a}{b-a} + \frac{\frac{1}{n_E}\sum_{l=1}^{n_E}\mu_{ji}^{(l)} - a}{b-a} = \frac{\frac{1}{n_E}\sum_{l=1}^{n_E}\left(\mu_{ij}^{(l)} + \mu_{ji}^{(l)}\right) - 2a}{b-a} = \frac{1-2a}{b-a} = 1 .$$

If $\frac{1}{n_E}\sum_{l=1}^{n_E}\mu_{ij}^{(l)} \geq b$, then analogous to the previous case one can make sure that this statement is correct. □

Another approach to aggregation of individual preferances can be the use of various kinds of operators of information aggregation. Let's examine the most reasonable and often used in practice family of aggregation operators. Ordered weighted averaging operator (OWA operator) was suggested by R. Yager in the work [Yager, 1988] and later more studied and characterized in [Yager, 1993]. OWA operator is commutative, idempotent, continuous, steady, neutral, equilibrated and stable relatively to lineal transformations but in general case is nonasociative. OWA operator accepts values from interval between the values of operators $Min(\cdot)$ and $Max(\cdot)$. Fundamental aspect of OWA operator is reordering of arguments in accordence with an importance (signification) of their values. R. Yager [Yager, 1993] defined the induced ordered weighted averaging operator (OWA operator) as the generalization of OWA operator for a case when information about competence of experts in form of crisp weight number is available. The same fuzzy principle of majority [Yager, 1994] is suggested to be used for calculation of weight numbers of OWA operator.

When using OWA operator for aggregation of individual expert preferances the following result is fair.

Statement 2 [Chiclana, 2003, p. 74]. Let Q is non-decreasing linguistic quantifier with such data as (a,b) that $a+b=1$. Then OWA operator managed with the quantifier Q retains the property of additivity.

For building an indirect collective ranging of alternatives on the basis of fuzzy collective preference we use the method suggested in the work [Скофенко, 1983]. It consists in the fact that on the basis of the available matrix of preferences $P = (\mu_{ij})_{i,j=1,\dots,n_A}$ it is possible to define the judgements of truth (assurance) of more difficult propositions concerning the alternatives, that is of the following propositions ω_{ik} , $k = 0,\dots,n_A - 1$, $\forall\, i \in N_A$:

ω_{ik} = "alternative a_i and is better than k alternative from set A".

If in the quality of t-norm, t-conorm and investor occur $\min\{\cdot,\cdot\}$, $\max\{\cdot,\cdot\}$, $1-\cdot$ then determination of truth degree of proposition ω_{ik} comes to the following rule [Скофенко, 1983]:

$$\mu(\omega_{ik}) = \begin{cases} 1-\mu_{ip_1}, \ if \ \ k=0, \\ \min\{\mu_{ip_k}, 1-\mu_{ip_k+1}\}, \ if \ \ 0 < k \leq n_A - 2, \\ \mu_{ip_{n_A}-1}, \ if \ \ k = n_A - 1, \end{cases}$$

where $\mu_{ip_k} \geq \mu_{ip_{k+1}}$, $k = 1,\dots,n_A - 1$.

Method of fuzzy collective ordinal judgement determination on the basis of fuzzy expert ordinal judgements

Rather wide spread procedure of expert information gaining is direct ranking of alternatives. Expert is proposed all set of alternatives for judgement and he is proposed to put them in order according to preferance. Direct ranking of alternatives can be realized by different ways [Литвак, 1996]. But in general case ordinal judegements given by an expert can be fuzzy. In a quality of fuzzy ordinal judgement can be the following fuzzy propositions of an expert:

- place (rank) of alternative a_i is nearly r_i;
- a_i is nearly within the limits from $r_i^{(1)}$ to $r_i^{(2)}$;
- fuzzy propositions which contain linguistic variable “rank”.

We formalize for our problem first two fuzzy judgements in form of traingular fuzzy number and trapezoidal fuzzy number accordingly.

$$\mu_r(x) = \begin{cases} 0, & if \ \ x < r - \dfrac{n_A}{10}, \\ 10 \cdot \dfrac{x-r}{n_A} + 1, & if \ \ r - \dfrac{n_A}{10} \le x < r, \\ 10 \cdot \dfrac{r-x}{n_A} + 1, & if \ \ r \le x < r + \dfrac{n_A}{10}, \\ 0, & if \ \ x \ge r + \dfrac{n_A}{10}. \end{cases}$$

$$\mu_{r_1 r_2}(x) = \begin{cases} 0, & if \ \ x < r_1 - \dfrac{n_A}{10}, \\ 10 \cdot \dfrac{x-r_1}{n_A} + 1, & if \ \ r_1 - \dfrac{n_A}{10} \le x < r_1, \\ 1, & if \ \ r_1 \le x < r_2, \\ 10 \cdot \dfrac{r_2-x}{n_A} + 1, & if \ \ r_2 \le x < r_2 + \dfrac{n_A}{10}, \\ 0, & if \ \ x \ge r_2 + \dfrac{n_A}{10}. \end{cases}$$

As it is noted in the work [Рыжов, 1998], it is easier for specialists in the applied problems where the expert judgements are widely used to formulate them in the terms of natural language. Such propositions of an expert are possible to formalize through the linguistic variable which is described by the tuple $\langle X, T(X), U, G, M \rangle$. Here X is the name of linguistic variable which reflects some obect; $T(X)$ is the set of values or terms of this variable which are the names of fuzzy variables; U is a set, which is the branch of terms definition; G is syntactic procedure (grammar), which describes the process of creation of set elements $T(X)$ of new values of linguistic variable; M is semantic procedure which allows to ascribe to each new meaning of linguistic variable some semantics by means of formation of corresponding fuzzy set. For our case: X = “rank”; $T(X)$ = {“high”, “middle”, “low”}; $U = [1, n_A]$; G = {“very”, “more or less”, “not”, “and”, “or”}; as the semantic rules we use the above mentioned rules for logic connection and negation. Membership functions of the corresponding terms can be defined in the following way:

$$\mu_{high}(x)=\begin{cases}0, & if\ \ x<1,\\ 1, & if\ \ 1\le x<\dfrac{n_A}{10}+1,\\ 10\cdot\dfrac{n_A-3x+3}{7n_A}, & if\ \ \dfrac{n_A}{10}+1\le x<\dfrac{n_A}{3}+1,\\ 0, & if\ \ x\ge\dfrac{n_A}{3}+1.\end{cases}$$

$$\mu_{middle}(x)=\begin{cases}0, & if\ \ x<\dfrac{n_A}{4},\\ \dfrac{4x-n_A}{n_A}, & if\ \ \dfrac{n_A}{4}\le x<\dfrac{n_A}{2},\\ \dfrac{3n_A-4x}{n_A}, & if\ \ \dfrac{n_A}{2}\le x<\dfrac{3}{4}n_A,\\ 0, & if\ \ x\ge\dfrac{3}{4}n_A.\end{cases}$$

$$\mu_{low}(x)=\begin{cases}0, & if\ \ x<\dfrac{2}{3}n_A,\\ 10\cdot\dfrac{3x-2n_A}{7n_A}, & if\ \ \dfrac{2}{3}n_A\le x<\dfrac{9}{10}n_A,\\ 1, & if\ \ \dfrac{9}{10}n_A\le x\le n_A,\\ 0, & if\ \ x>n_A.\end{cases}$$

Thus an alternative of paired comparison of alternatives (in literature such method of expert judgements giving got the name of giving "object-object"[Cook, 1983]) can be the method "object-rank" of expert judgements giving. As a result of such approach the experts evidently or implicitly form their individual judgements in the form of matrixes $P_l=(\mu_{ik}^{(l)})_{i,k=1,\ldots,n_A}$, $l=1,\ldots,n_E$, elements of which show the truth degree of proposition

ω_{ik} = "alternative a_i has rank k".

On the stage of expert information aggregation on the basis of available fuzzy ordinal individual judgements we define the truth degree of the following fuzzy proposition:

$\omega_{ik}^{(C)}$ = "most experts consider that alternative a_i has rank k".

Truth degree of such fuzzy proposition is calculated according to the following rule:

$$\mu_{ik}^{(C)}=\begin{cases}0, & if\ \ \dfrac{1}{n_E}\sum_{l=1}^{n_E}\mu_{ik}^{(l)}\le 0.3,\\ \dfrac{2}{n_E}\sum_{l=1}^{n_E}\mu_{ik}^{(l)}-0.6, & if\ \ 0.3<\dfrac{1}{n_E}\sum_{l=1}^{n_E}\mu_{ik}^{(l)}<0.8,\\ 1, & if\ \ 0.8\le\dfrac{1}{n_E}\sum_{l=1}^{n_E}\mu_{ik}^{(l)}\le 1,\end{cases}$$

where $\mu_{ik}^{(l)}=\mu_l(\omega_{ik})$, $\mu_l(\cdot)$ is the membershop function of corresponding fuzzy rank, given ny the l st expert.

By collective fuzzy ranking of the set of alternatives A we shall understand the set of all fuzzy subsets A_i, $i=1,\ldots,n_A$, which are determined by the values $\mu_{ik}^{(C)}$, $k=1,\ldots,n_A$, and correspondingly with the following membership functions [Скофенко, 1983]:

$$\mu_{A_i}(k) = \frac{\mu_{ik}^{(C)}}{\max\limits_{k=1,\ldots,n_A} \mu_{ik}^{(C)}}. \tag{4}$$

Approaches to the definition of strict collective ranking of alternatives on the basis of fuzzy collective ordinal judgement

It is known [Скофенко, 1983], that in the case when the matrix of fuzzy preference is the matrix of fuzzy tournament then the corresponding fuzzy ranking has the property of prominence in the sense of fuzzy set promonence. At the cut A_i, which comes into fuzzy ranking according to all values of membership degree, the single segment in the set of crisp ranks will be put in correspondence to each alternative. If the matter is about the fuzzy individual rankings and if it is impossible to define crisp resulting rankings on the basis of α-cut, the experts are proposed to overview their judgements, after which the above described approach is used again.

One can use the following approach for definition of crisp strict ranking which is in some sense “the closest” to the fuzzy collective ranking. To the crisp ranking of alternative by giving “object-rank” expert judgements evidently corresponds matrix $X = (x_{ik})_{i,k=1,\ldots,n_A}$, elements of which satisfiy the conditions $x_{ik} \in \{0,1\}$, $\sum_{i=1}^{n_A} x_{ik} = \sum_{k=1}^{n_A} x_{ik} = 1$, $i,k = 1,\ldots,n_A$. If as the proximity measure between fuzzy rankings Hamming distance is taken between the corresponding matrixes, then the following arrangement is justified:

$$\sum_{i=1}^{n_A}\sum_{k=1}^{n_A} | x_{ik} - \mu_{ik}^{(C)} | \to \min,$$

$$\sum_{i=1}^{n_A} x_{ik} = \sum_{k=1}^{n_A} x_{ik} = 1, \; x_{ik} \in \{0,1\}, \; i,k = 1,\ldots,n_A.$$

Example

Let each of the expert group $\{e_\}, e_2, e_3, e_4\}$ makes direct ranking of seven alternatives of set A. Result of carried out judgements is shown in table 1.

Table 1. *Fuzzy ordinal expert judgements*

	e_1	e_2	e_3	e_4
a_1	high	nearly 1	average	nearly 2
a_2	nearly 3	high	not low and not very high	average
a_3	average or low	not very high	low	very low
a_4	not low	small	nearly 2	Very high
a_5	Not very high	average	not very low	nearly 4
a_6	Within limit of [5,7]	not high	high	not very high
a_7	low	nearly 6	average	Not high

We calculate value $\mu_{ik}^{(C)}$ for $i,k = 1,\ldots,7$ on the basis of expert rankings (table 1) and put them into the table 2.

Table 2. *Collective fuzzy ordinal judgement*

	1	2	3	4	5	6	7
a_1	**0.543**	0.522	0.002	0	0	0	0
a_2	0	0.189	**0.838**	0.4	0	0	0
a_3	0	0	0.236	0.257	0.125	1	**1**
a_4	0.543	**0.733**	0.064	0	0	0	0
a_5	0	0.138	0.879	**1**	0.593	0.067	0
a_6	0	0.067	0.379	0.543	**0.9**	0.9	0.9
a_7	0	0	0.155	0.257	0.216	**0.808**	0.543

Then according to the equation (4) we calculate value $\mu_{A_i}(k)$, $i,k = 1,\ldots,7$. Fuzzy ranking, given in the form of membership function is showed on the figure 1.

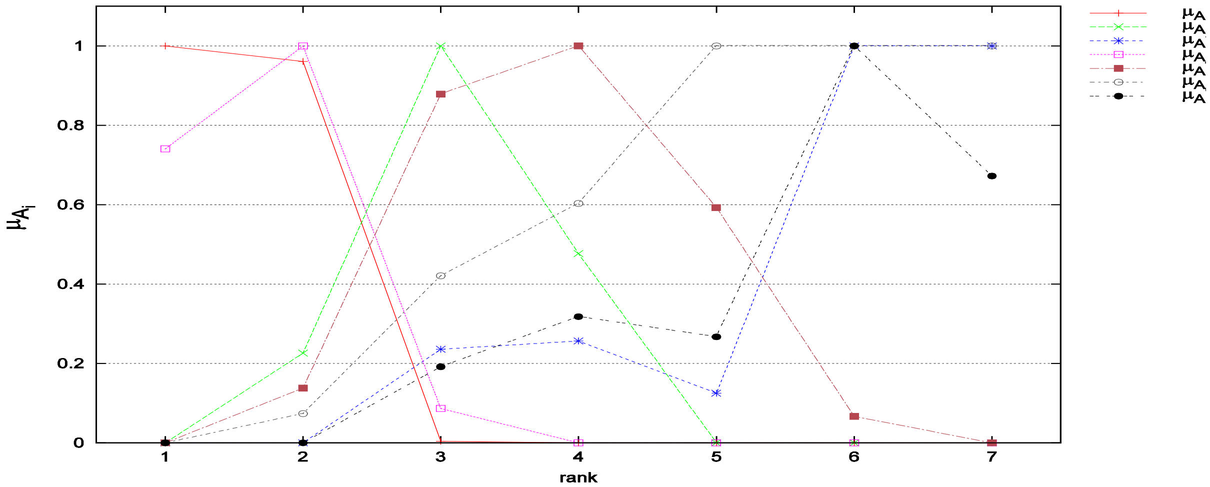

Figure 1. *Fuzzy collective ranking of the set of alternatives A*

On the basis of cut of fuzzy collective ranking according to the highest membership degree as the solution of our model problem we get crisp strict ranking of alternatives to which corresponds permutation of alternative indexes $(1,4,2,5,6,7,3)$.

Conclusions

Indirect approach developed in this work can serve as an alternative to the direct approach, realized by the authors before in the work [Антосяк, 2010].

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Bibliography

[Тоценко, 2006] Тоценко В. Г. Об унификации алгоритмов организации экспертиз // Проблемы правовой информатизации, 2006, 2 (12). – С. 96-101.

[Kacprzyk, 1985i] Kacprzyk J. Group decision making with a fuzzy majority via linguistic quantifiers. part i: A consensory-like pooling // Cybernetics and Systems, 1985, 16. – P. 119-129.

[Kacprzyk, 1985ii] Kacprzyk J. Group decision making with a fuzzy majority via linguistic quantifiers. part ii: A competitive-like pooling // Cybernetics and Systems, 1985, 16. – P. 131-144.

[Yager, 1988] Yager R. R. On ordered weighted averaging aggregation operators in multicriteria decision making // Systems, Man and and Cybernetics, Part A: Systems and Humans, 1988, 18. – P. 183-190.

[Yager, 1993] Yager R. R. Families of owa operators // Fuzzy Sets and Systems, 1993, 59, 2. – P. 125-148.

[Yager, 1994] Yager R. R. Quantifier guided aggregation using owa operators // International Journal of Intelligent Systems, 1994, 11. – P. 49-73.

[Chiclana, 2003] F. Chiclana, F. Herrera, E. Herrera-Viedma, L. Martinez. Anote on the reciprocity in the aggregation of fuzzy preference relations using owa operators // Fuzzy Sets and Systems, 2003, 137. – P. 71-83.

[Скофенко, 1983] Скофенко А. В. Применение нечеткой логики при ранжировании объектов методом парных сравнений // Кибернетика, 1983, 3. – С. 116-118.

[Литвак, 1996] Литвак Б. Г. Экспертные оценки и принятие решений. И-во «Патент», Москва 1996. – 271 с.

[Рыжов, 1998] Рыжов А. П. Элементы теории нечетких множеств и измерения нечеткости. И-во «Диалог-МГУ»,
1998. – 116 с.

[Cook, 1983] W. D. Cook, L. Seiford, S. Warner. Preference ranking models: Conditions for equivalence // Journal of Mathematical Sociology, 1983, 9. – P. 125-127.

[Антосяк, 2010] Антосяк П. П. Узагальнення медіанного підходу на випадок нечітких індивідуальних переваг // Вісник Київського університету. Серія: фіз.-мат. науки, 2010, 2. – С. 81-86.

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