A HYBRID APPROACH FOR ASSESSING PROBLEM SOLVING SKILLS UNDER FUZZY CONDITIONS

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Abstract: *Volumes of research have been written about problem solving, which is one of the most important components of the human cognition affecting the progress of the human society for ages. In this work a hybrid method is applied for the assessment of a student group’s problem solving skills with qualitative grades (i.e. under fuzzy conditions). Namely, soft sets are used as tools for a parametric assessment of the group’s performance, the calculation of the GPA index and the Rectangular Fuzzy Assessment Model are applied for evaluating the group’s qualitative performance, grey numbers are used as tools for assessing the group’s mean performance and neutrosophic sets are utilized when the teacher is not sure about the individual grades obtained by some (or all) students of the group.*

Keywords: *Problem Solving, Assessment Methods, GPA Index, Fuzzy Sets and Logic, Rectangular Fuzzy Assessment Model, Grey Numbers, Neutrosophic Sets, Soft Sets.*

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Introduction

Volumes of research have been written about problem solving (PS), which is one of the most important components of the human cognition affecting the progress of the human society for ages. In [Voskoglou, 2011] we have examined the role of the problem in learning mathematics and we have attempted a review of the evolution of research on PS in mathematics education from the time of Polya until today.

Polya laid during the 50's and 60's the foundation for exploration in *heuristics* for PS, being the first who described them in a way that they could be taught. The failure of the introduction of the "New Mathematics" to school education placed the attention of specialists during the 80's on the use of the problem as a tool and motive to teach and understand better mathematics. A framework was created describing the PS process and reasons for success or failure in PS, which was depicted in Schoenfeld's *Expert Performance Model (EPM)* for PS [Schoenfeld, 1980]. The steps of the PS process in this model (see Figure 1) are the *analysis* (S_1) of the problem, the *design* (S_2) of its solution through the *exploration* (S_3), the *implementation* (S_4) and the *verification* (S_5) of the solution.

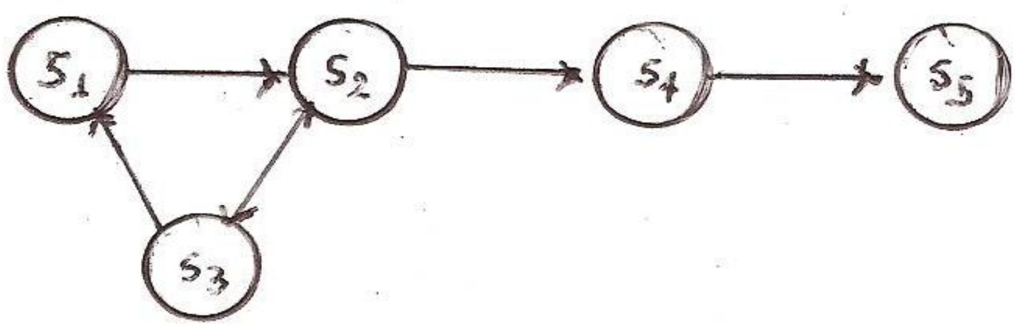

Figure 1. The "flow-diagram" of EPM

While early work on PS focused on describing the PS process, more recent investigations during the 2000's focused on identifying attributes of the problem solver that contribute to successful PS. Carlson and Bloom drawing from the large amount of literature related to PS developed a broad taxonomy to characterize major PS attributes that have been identified as relevant to PS success. This taxonomy gave genesis to their *Multidimensional PS Framework (MPSF)* [Carlson & Bloom, 2005], which includes the following steps: *Orientation, Planning, Executing* and *Checking* (Figure 2).

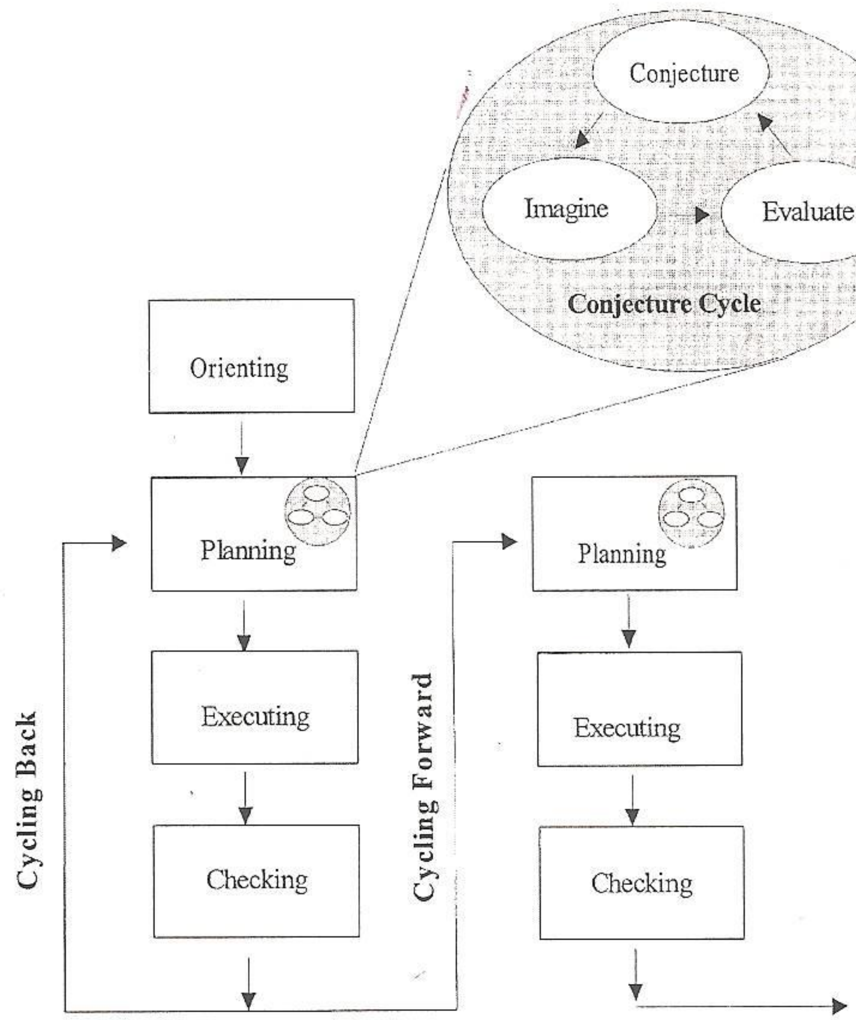

Figure 2. The "flow-diagram" of MPSF

It was observed that, when contemplating various solution approaches during the planning step of the PS process, the solvers were at times engaged in a *conjecture-imagine-evaluate* sub-cycle. It is of worth noting that a careful inspection of the two PS models shows that the steps of MPSF are in one-to-one correspondence to the steps of Schoenfeld's EPM, with S_1 corresponding to orientation, S_2 to planning, S_3 to the conjecture-imagine-evaluate sub-cycle, S_4 to executing and S_5 to checking.. However, there exists a basic qualitative difference between the two models: While in MPSF the emphasis is turned to the solver's behavior and required attributes, the EPM is oriented towards the PS process itself describing the proper heuristic strategies that may be used at each step of the PS process.

Schoenfeld, after a many years effort and research for building a theoretical framework providing rigorous explanations on how and why people during the PS process make the choices they did, concluded that the PS process, as well as many other human activities like cooking, teaching a lesson and even a brain surgery, are all examples of a *goal-directed behavior.* Thus, the individual's "acting in the moment" can be explained and modelled by a theoretical architecture in which knowledge, goals, orientations and decision-making are involved. The different individuals' decision choices can be seen as modelled by "expected-value" computations, where the quantities are the "subjective values" assigned by the individuals. In fact, the expected value of a decision equals the probability for the decision to be correct multiplied by the value of its profit minus its cost. But from each person's subjective point of view the value of a decision's profit is different and therefore its expected value is also different. That explains why different people will decide differently, because the subjective values they assigned are different. Schoenfeld argues that, once you understand an individual's orientations, you can see how the individual priorizes goals and outcomes and therefore you can model the possible courses of his action. Thus, when you understand how something skilful is done, you can help the others to do it successfully [Schoenfeld, 2010].

Quality is a desirable characteristic of all human activities. This makes assessment one of the most important components of the processes connected to those activities. Assessment takes place in two ways, either with the help of numerical or with the help of qualitative grades. When numerical grades are used, standard methods are applied for the overall assessment of the skills of a group of objects participating in a certain activity, like the calculation of the mean value of all the individual scores. The use of qualitative grades is usually preferred when more elasticity is desirable (as it frequently happens in case of student assessment), or when no exact numerical data are available. In this case, assessment methods based on principles of *fuzzy logic (FL)* are frequently used.

The present author has developed in earlier works several methods for the assessment of human/machine performance under fuzzy conditions including the measurement of *uncertainty* in fuzzy systems, the use of the *Center of Gravity (COG) defuzzification technique*, the use of *fuzzy numbers (FNs)* and of *grey numbers (GNs)*, etc. All these methods are reviewed in [Voskoglou, 2019a]. Recently, the same author developed a model for parametric assessment that uses *soft sets (SSs)* as tools [Voskoglou, 2022] and he also used *neutrosophic sets (NSs)* for student assessment when the teacher is not absolutely sure for the grades assigned to students [Voskoglou, et al., 2022].

In this paper a hybrid method is presented for the assessment of student PS skills with qualitative grades (therefore under fuzzy conditions) using the GPA index, the *Rectangular Fuzzy Assessment Model (RFAM)*, GNs, NSs and SSs as tools. The rest of the paper is organized as follows: The next Section contains basic information about GNs, NSs, SSs, GPA index and RFAM, needed for the understanding of the paper. The hybrid assessment method is developed in the third Section and the paper closes with a short discussion about future research perspectives and the final conclusions.

Mathematical Background

Fuzzy Sets and Logic

Zadeh, in order to deal with partial truths, introduced in 1965 the concept of *fuzzy set (FS)* as follows [Zadeh, 1965]:

Definition 1: Let U be the universe, then a FS F in U is of the form

$$F = \{(x, m(x)): x \in U\} \tag{1}$$

In equation (1) m: $U \rightarrow [0,1]$ is the *membership function* of F and m(x) is called the *membership degree* of x in F**.** The greater m(x), the more x satisfies the property of F. A crisp subset F of U is a FS in U with membership function such that m(x)=1 if x belongs to F and 0 otherwise.

Based on the concept of FS Zadeh developed the infinite-valued FL [Zadeh, 1973], in which truth values are modelled by numbers in the unit interval [0, 1]. FL is an extension of the classical *bivalent logic (BL)* embodying the Lukasiewicz’s “Principle of Valence”. According to this principle propositions are not only either true or false (according to the Aristotle’s principle of the “Excluded Middle”), but they can have intermediate truth-values too.

It was only in a second moment that FS theory and FL were used to embrace *uncertainty* modelling [Zadeh, 1978, Dubois & Prade 2001]. This happened when membership functions were reinterpreted as possibility distributions. *Possibility theory* is an uncertainty theory devoted to the handling of incomplete information [Dubois & Prade 2006]. Zadeh articulated the relationship between possibility and *probability*, noticing that what is probable must preliminarily be possible. For general facts on FSs and the connected to them uncertainty we refer to the book [Klir & Folger, 1988].

Neutrosophic Sets

Following the introduction of FSs, various generalizations and other related to FSs theories have been proposed enabling, among others, a more effective management of all types of the existing in real world uncertainty. A brief description of the main among those generalizations and theories can be found in [Voskoglou, 2019b].

Atanassov added in 1986 to Zadeh’s membership degree the *degree of non-membership* and introduced the concept of *intuitionistic fuzzy set (IFS)* [Atanassov, 1986] as the set of the ordered triples

$$A = \{(x, m(x), n(x)): x \in U, 0 \leq m(x) + n(x) \leq 1\} \tag{2}$$

Smarandache, motivated by the various neutral situations appearing in real life - like <friend, neutral, enemy>, <positive, zero, negative>, <small, medium, high>, <male, transgender, female>, <win, draw, defeat>, etc. – introduced in 1995 the degree of *indeterminacy/neutrality* of the elements of the universal set U in a subset of U and defined the concept of *neutrosophic set (NS)* [Smarandache, 1998]. The term neuttrosophic is the result of a synthesis of the words “neutral” and “sophia” meaning in Greek “wisdom”. In this work we need only the simplest version of the concept of NS, which is defined as follows:

Definition 2: A *single valued NS (SVNS)* A in U is of the form

$$A = \{(x,T(x),I(x),F(x)): x \in U,\ T(x),I(x),F(x) \in [0,1],\ 0 \leq T(x)+I(x)+F(x) \leq 3\} \quad (3)$$

In (3) T(x), I(x), F(x) are the degrees of *truth* (or membership), indeterminacy and *falsity* (or non-membership) of x in A respectively, called the *neutrosophic components* of x. For simplicity, we write A<T, I, F>.

For example, let U be the set of the players of a basketball team and let A be the SVNS of the good players of U. Then each player x of U is characterized by a *neutrosophic triplet* (t, i, f) with respect to A, with t, i, f in [0, 1]. For instance, x(0.7, 0.1, 0.4) ∈ A means that there is a 70% belief that x is a good player, a 10% doubt about it and a 40% belief that x is not a good player. In particular, x(0,1,0) ∈ A means that we do not know absolutely nothing about x’s affiliation with A.

In an IFS the indeterminacy coincides by default to 1- T(x) – F(x). Also, in a FS is I(x)=0 and F(x) = 1 – T(x), whereas in a crisp set is T(x)=1 (or 0) and F(x)= 0 (or 1). In other words, crisp sets, FSs and IFSs are special cases of SVNSs.

When the sum T(x) + I(x) + F(x) of the neutrosophic components of x ∈ U in a SVNS A on U is <1, then x leaves room for *incomplete* information, when is equal to 1 for *complete* information and when is greater than 1 for *paraconsistent* (i.e. contradiction tolerant) information. A SVNS may contain simultaneously elements leaving room to all the previous types of information. For general facts on SVNSs we refer to [Wang et al., 2010].

Summation of neutrosophic triplets is equivalent to the neutrosophic union of sets. That is why the neutrosophic summation and implicitly its extension to neutrosophic scalar multiplication can be defined in many ways, equivalently to the known in the literature neutrosophic union operators [Smartandache, 2016]. Here, writing the elements of a SVNS A in the form of neutrosophic triplets we define addition and scalar product in A as follows:

Let (t_1, i_1, f_1), (t_2, i_2, f_2) be in A and let k be appositive number. Then;

- The *sum* $(t_1, i_1, f_1) + (t_2, i_2, f_2) = (t_1+ t_2, i_{1+} i_2, f_{1+} f_2)$ (4)

- The *scalar product* $k(t_1, i_1, f_1) = (kt_1, k\, i_1, kf_1)$ (5)

Soft Sets

A disadvantage connected to the concept of FS is that there is not any exact rule for defining properly the membership function. The methods used for this are usually empirical or statistical and the definition of the membership function is not unique depending on the "signals" that each observer receives from the environment, which are different from person to person. For example, defining the FS of "tall men" one may consider as tall all men having heights more than 1.90 meters and another all those having heights more than 2 meters. As a result, the first observer will assign membership degree 1 to men of heights between 1.90 and 2 meters, in contrast to the second one, who will assign membership degrees <1. Consequently, analogous differences is logical to appear for all the other heights. The only restriction, therefore, for the definition of the membership function is to be compatible to the common sense; otherwise the resulting FS does not give a reliable description of the corresponding real situation. This could happen for instance, if in the FS of "tall men", men with heights less than 1.60 meters have membership degrees ≥0.5.

The same difficulty appears to all generalizations of FSs in which membership functions are involved (e.g. IFSs, NSs, etc.). For this reason, the concept of *interval-valued FS (IVFS)* [Dubois & Prade, 2005] was introduced in 1975, in which the membership degrees are replaced by sub-intervals of the unit interval [0, 1]. Alternative to FS theories were also proposed, in which the definition of a membership function is either not necessary (*grey systems/GNs* [Deng, 1982]), or it is overpassed by considering a pair of sets which give the lower and the upper approximation of the original crisp set (*rough sets* [Pawlak, 1991]).

Molodstov, in order to tackle the uncertainty in a parametric manner, initiated in 1999 the concept of *soft set (SS)* as follows [Molodstov, 1999]:

Definition 3: Let E be a set of parameters, let A be a subset of E, and let f be a map from A into the power set P(U) of all subsets of the universe U. Then the SS (f, A) in U is defined to be the set of the ordered pairs

$$(f, A) = \{(e, f(e)): e \in A\} \tag{6}$$

The term "soft" is due to the fact that the form of (f, A) depends on the parameters of A. For example, let U= {C_1, C_2, C_3} be a set of cars and let E = {e_1, e_2, e_3} be the set of the parameters e_1=cheap, e_2=hybrid (petrol and electric power) and e_3= expensive. Let us further assume that the cars C_1, C_2 are cheap, C_3 is expensive and C_2, C_3 are hybrid cars. Then, a map f: E $\rightarrow$ P(U) is defined by f(e_1)={C_1, C_2}, f(e_2)={C_2, C_3} and f(e_3)={C_3}. Therefore, the SS (f, E) in U is the set of the ordered pairs (f, E) = {(e_1, {C_1, C_2}), (e_2, {C_2, C_3}, (e_3, {C_3}}.

A FS in U with membership function y = m(x) is a SS in U of the form (f, [0, 1]), where f(α)={x$\in$U: m(x)$\geq$α} is the corresponding $\alpha - cut$ of the FS, for each α in [0, 1]. For general facts on SSs we refer to [Maji et al., 2003].

Obviously, an important advantage of SSs is that, by using the parameters, they pass through the need of defining membership functions. The theory of soft sets has found many and important applications to several sectors of the human activity like decision making, parameter reduction, data clustering and data dealing with incompleteness, etc. One of the most important steps for the theory of soft sets was to define mappings on soft sets, which was achieved by A. Kharal and B. Ahmad and was applied to the problem of medical diagnosis in medical expert systems [Kharal & Ahmad, 2011]. But fuzzy mathematics has also significantly developed at the theoretical level providing important insights even into branches of classical mathematics like algebra, analysis, geometry, topology etc.

We ought to note, however, that, despite the fact that IFSs and SSs have already found many and important applications, there exist reports in the literature disputing the significance of these concepts and considering them as redundant, representing in an unnecessarily complicated way standard fixed-basis set theory [Garcia & Rodabaugh, 2005, Shi & Fan, 2019]. In the Abstract of [Shi & Fan, 2019], for example, one reads: "In particular, a soft set on X with a set E of parameters actually can be regarded as a 2E -fuzzy set or a crisp subset of E × X [the correct is E x P(X)]. This shows that the concept of (fuzzy) soft set is redundant". We completely disagree with this way of thinking. Adopting to it, one could claim that, since a FS A in X is a subset of the Cartesian product X x m(X), where m is the membership function of A, the concept of FS is redundant!

Grey Numbers

Approximate data are frequently used nowadays in many problems of everyday life, science and engineering, because many constantly changing factors are usually involved in large and complex systems. Deng introduced in 1982 the *grey system (GS)* theory as an alternative to the theory of FSs for tackling such kind of data [Deng, 1982]. A GS is understood to be a system that lacks information such as structure message, operation mechanism and/or behaviour document. The GS theory, which has been mainly developed in China, has recently found many important applications [Deng, 1989].

An interesting application of the closed intervals of real numbers is their use in the GS theory for handling approximate data. In fact, a numerical interval I = [x, y], with x, y real numbers, x<y, can be considered as representing a real number with known range, whose exact value is unknown. The closer x to y, the better I approximates the corresponding real number. When no other information is given about this number, it looks logical to consider as its representative approximation the real value

$$V(I) = \frac{x+y}{2} \tag{7}$$

[Moore et al., 1995] introduced the basic arithmetic operations on closed real intervals. In the present work we shall make use only of the addition and scalar product defined as follows: Let $I_1 = [x_1, y_1]$ and $I_2 = [x_2, y_2]$ be closed intervals, then their *sum* $I_1 + I_2$ is the closed interval

$$I_1 + I_2 = [x_1+ x_2, y_1+ y_2] \tag{8}$$

Further, if k is a positive number then the *scalar product* kI_1 is the closed interval

$$kI_1 = [kx_1, ky_1] \tag{9}$$

When the closed real intervals are used for handling approximate data, are usually referred as *grey numbers (GNs)*. A GN [x, y], however, may also be connected to a *whitenization function* f: [x, y] → [0, 1], such that, ∀ a ∈ [x, y], the closer f(a) to 1, the better a approximates the unknown number represented by [x, y].

We close this subsection with the following definition, which will be used in the assessment method that will be presented later in this work.

Definition 4: Let $I_1, I_2, \ldots, I_n$ be a finite number of GNs, $n \geq 2$, then the *mean value* of these GNs is defined to be the GN

$$I = \frac{1}{n}(I_1 + I_2 + \ldots + I_k) \tag{10}$$

GPA Index and the Rectangular Fuzzy Assessment Model

The calculation of the *Grade Point Average (GPA) Index* is a classical method, very popular in USA and other western countries, for evaluating a group's *qualitative performance*, when greater coefficients are assigned to the higher grades. For this, let n be the total number of the objects of the group under assessment and let n_X be the number of the group's objects obtaining the grade X, X =A, B, C, D, F, where A=excellent, B=very good, C=good, D=mediocre and F=unsatisfactory. Then, the GPA index is calculated by the formula

$$\text{GPA} = \frac{0n_F + n_D + 2n_C + 3n_B + 4n_A}{n} \tag{11}$$

[Voskoglou, 2017] (Chapter 6, p. 125)

In the worst case ($n=n_F$) equation (11) gives that GPA=0, whereas in the best case ($n=n_A$) it gives that GPA=4. We have in general, therefore, that 0≤GPA≤4, which means that values of GPA≥2 indicate a satisfactory qualitative performance.

Setting $y_1 = \frac{n_F}{n}$, $y_2 = \frac{n_D}{n}$, $y_3 = \frac{n_C}{n}$, $y_4 = \frac{n_B}{n}$ and $y_5 = \frac{n_A}{n}$, equation (11) can be written as

$$\text{GPA} = y_2 + 2y_3 + 3y_4 + 4y_5 \tag{12}$$

Voskoglou developed a fuzzy model for representing mathematically the process of learning a subject matter in the classroom [Voskoglou, 2000]. Later, considering a student class as a fuzzy system, he calculated the existing in it *total possibilistic uncertainty* for assessing the student mean performance [Voskoglou, 2009]. Subbotin et al., based on Voskoglou's model, adapted properly the *Center of Gravity (COG) defuzzification technique* for use as an assessment method of student learning skills [Subbotin et al., 2004]. Since then, Subbotin and Voskoglou applied, jointly or separately, the COG technique, termed by them as the *Rectangular Fuzzy Assessment Model (RFAM)*, in many other types of assessment problems; e.g. see [Voskoglou, 2017] (Chapter 6).

There is a commonly used in FL approach to represent the fuzzy data by the coordinates (x_c, y_c) of the COG of the level's area between the graph of the corresponding membership function and the OX axis [Van Broekhoven & Debaets, 2006]. In our case, keeping the same notation as for the GPA index, it can be shown that the coordinates of the COG are calculated by the formulas

$$x_c = \frac{1}{2}(y_1+3y_2+5y_3+7y_4+9y_5) \tag{13}$$

$$y_c = \frac{1}{2}(y_1^2+y_2^2+y_3^2+y_4^2+y_5^2) \tag{14}$$

[Voskoglou, 2019a] (Section 4)

It can be also shown the following result [Voskoglou, 2019a] (Section 4**):**

Assessment Criterion:

- Between two groups, the group with the greater x_c demonstrates the better performance.
- For two groups with the same value of x_c, if $x_c \geq 2.5$ the group with the greater value of y_c performs better, and if $x_c < 2.5$ the group with the lower value of y_c performs better.

Combining equations (12) and (13) one finds that $x_c = \frac{1}{2}(2GPA + 1)$ or

$$x_c = GPA + \frac{1}{2} \tag{15}$$

Thus, with the help of the first case of the previous criterion, one concludes that, if the GPA value of two student groups is different, then the RFAM and the GPA index give the same outcomes concerning the assessment of the qualitative performance of the two groups. If the GPA index, however, is the same for the two groups, then one MUST apply the RFAM to see which group performs better.

The Hybrid Assessment Model

A hybrid method is applied in this Section for the assessment of a student group's PS skills with qualitative grades. Namely, SSs are used as tools for a parametric assessment of the group's performance, the calculation of the GPA index and the RFAM are applied for evaluating the group's qualitative performance, GNs are used as tools for assessing the group's mean performance and NSs are used when the teacher is not sure about the individual grades assigned to some (or all) students.

Parametric Assessment Using Soft Sets

Assume that a mathematics teacher wants to assess the PS skills of a group U = {S_1, S_2, ….., S_n} of students. Let E = {A, B, C, D, E} be the set of parameters A=excellent, B=very good, C=good, D=mediocre and F=unsatisfactory. Assume further that the first four students of the group demonstrated excellent performance, the next five very good, the following 7 good, the next eight mediocre and the rest of them unsatisfactory performance. Let f be the map assigning to each parameter of E the subset of students whose performance was assessed by this parameter. Then, the overall student performance is represented mathematically by the SS

$$(f, E) = \{(A, \{S_1, S_2, S_3\}), (B, \{S_4, S_5,\dots, S_8\}), (C, \{S_9, S_{10}, \dots, S_{15}\}), (D, \{S_{16}, S_{17},\dots, S_{23}\}), (F, \{S_{24}, S_{25,\dots,} S_n\}) \tag{16}$$

The use of SSs enables also the representation of each student’s individual performance at each step of the PS process. In fact, let T_1=orientation, T_2=planning, T_3=conjecture-imagine-evaluate, T_4=executing and T_5=checking be the steps of the previously described MPSF. Set V = {T_1, T_2, T_3, T_4, T_5}, consider a particular student of U and define a map f: E$\rightarrow$Δ(V) assigning to each parameter of E the subset of V consisting of the steps of the PS process assessed by this parameter with respect to the chosen student. For example, the soft set

$$(f, E) = \{(A, \{T_1, T_3\}), (B, \{T_5\}), (C, \{T_4\}), (D, \{T_2\}), (F, \emptyset)\} \quad (17)$$

represents the profile of a student who demonstrated excellent performance at the steps of orientation and conjecture-imagine-evaluate, very good performance at the step of checking, good performance at the step of executing and mediocre performance at the step of planning (he/she faced difficulties, but he/she finally came through).

Use of the COG Technique and the RFAM for Assessing a Group's Qualitative Performance

The following example illustrates this method:

Example 1: The students of two classes obtained the following grades in a mathematical PS test: Class I: A=5 students, B=3, C=7, D=0, F=5, Class II: A=4, B=4, C=7, D=1, F=4. Which class demonstrated the better qualitative performance?

Solution: Equation (11) gives that $GPA_I = GPA_2 = \frac{43}{20}$. The RFAM model must be used, therefore, for comparing the two classes' qualitative performance. Thus, by equation (13) one gets that $x_{C_I} = x_{C_{II}} = = \frac{53}{20} > \frac{5}{2}$. But equation (14) gives that y_{C_I} = 54 and $y_{C_{II}}$ = 49, therefore, by the second case of the RFAM assessment criterion, one concludes that Class I demonstrated a better qualitative performance. Further, since GPA_I =

$GPA_2 = \frac{43}{20} > 2$, both groups demonstrated satisfactory qualitative performance.

Use of Grey Numbers for Evaluating a Group’s Mean Performance.

When the student individual assessment is realized with qualitative grades, a student group’s mean performance cannot be assessed with the classical method of calculating the mean value of the student scores. To overcome this difficulty, using the numerical climax 1-100 we assign to each of the student qualitative grades a closed real interval (GN), denoted for simplicity with the same letter, as follows: A = [85, 100], B = [75, 84], C = [60, 74], D = [50, 59] and F= [0, 49]. It is of worth noting that, although the GNs assigned to the qualitative grades satisfy generally accepted assessment standards, the previous assignment is not unique, depending on the teacher’s personal goals. For a more strict assessment, for example, the teacher could choose A = [90, 100], B = [80, 89], C = [70, 79], D = [60, 69], F= [0, 59], etc.

The estimation of a group’s mean performance with the help of the previously defined GNs is illustrated with the following example:

Example 2: Reconsider Example 1. Which class demonstrated the better mean performance?

Solution: Under the light of equation (10), it is logical to accept that the GNs $M_I = \frac{1}{20}$ (5A+3B+7C+0D+5F) and $M_{II} = \frac{1}{20}$ (4A+4B+7C+1D+4F) respectively can be used for estimating the two classes’ mean performance. Straightforward calculations with the help of equations (8) and (9) give that $M_I = \frac{1}{20}[1070, 1515] = [53.5, 75.75]$ and $M_{II} = \frac{1}{20}[1110, 1509] = [55.5, 75.45]$. Equation (7) gives, therefore, that $V(M_I) = 64.625$ and $V(M_{II}) = 64.75$. Thus, both classes demonstrated good (C) mean performance, with the mean performance of Class II being better.

Using Neutrosophic Sets for the Assessment

In many cases the teacher has doubts about the grades assigned to some (or all) students. In such cases the use of NSs is more appropriate for estimating the student group overall performance. This process is illustrated in the following example:

Example 3: Let $\{s_1, s_2, \ldots, s_{20}\}$ be a class of 20 students. The teacher of the class is not sure about the grades obtained by them in a test on mathematical PS, because some of the students did not give proper explanations about their solutions. The teacher decides, therefore, to characterize the students who demonstrated excellent performance in the test using neutrosophic triplets as follows: $s_1(1, 0, 0)$, $s_2(0.9, 0.1, 0.1)$, $s_3(0.8, 0.2, 0.1)$, $s_4(0.4, 0.5, 0.8)$, $s_5(0.4, 0.5, 0.8)$, $s_6(0.3, 0.7, 0.8)$, $s_7(0.3, 0.7, 0.8)$, $s_8(0.2, 0.8, 0.9)$, $s_9(0.1, 0.9, 0.9)$, $s_{10}(0.1, 0.9, 0.9\}$ and for all the other students (0, 0, 1). This means that the teacher is absolutely sure that s_1 demonstrated excellent performance, 90% sure that s_2 demonstrated excellent performance too, but at the same time has a 10% doubt about it and also a 10% belief that s_2 did not demonstrated excellent performance, etc. For the last 10 students the teacher is absolutely sure that they did not demonstrate excellent performance. What should be the teacher's conclusion about the class's mean mathematical level in this case?

Solution: It is logical to accept that the class's mean mathematical level can be estimated by the neutrosophic triplet $\frac{1}{20}$[(1, 0, 0)+(0.9, 0.1, 0.1)+(0.8, 0.2, 0.1)+2(0.4, 0.5, 0.8)+2(0.3, 0.7, 0.8)+(0.2, 0.8, 0.9)+2(0.1, 0.9, 0.9)+10(0, 0, 1)], which by equations (8) and (9) is equal to $\frac{1}{20}$(4.5, 5.3, 16.3) = (0.225, 0.265, 0.815). This means that a random student of the class has a 22.5 % probability to be an excellent student, however, there exist also a 26.5% doubt about it and an 81.5% probability to be not an excellent student. Obviously this conclusion is characterized by inconsistency.

The teacher could work in the same way by considering the NSs of students who demonstrated very good, good, mediocre and unsatisfactory performance in the test, thus obtaining analogous conclusions.

Discussion and Conclusions

A hybrid assessment method was applied in this work for assessing student PS skills under fuzzy conditions (with qualitative grades). The discussion performed leads to the following conclusions:

- SSs can be used for realizing a parametric assessment of the student group's overall performance
- The qualitative performance of a student group (where greater coefficients are assigned to the higher grades) can be measured

either by the classical method of calculating the GPA index, or by applying the RFAM, which is based on the COG defuzzification technique. When two groups have the same GPA index, however, then the RFAM model must be applied to find which group demonstrates the better performance.

- In case of using qualitative grades for assessing the student performance, the assessment of a student group's mean performance cannot be realized by the classical way of calculating the mean value of the student individual scores. The student mean performance in this case can be estimated by using GNs (closed real intervals).

- When the teacher has doubts for the grades obtained to some (or all) students, NSs can be used for assessing the overall performance of a student group.

Our experience from the present and earlier works implies that hybrid methods, like the previous one, give usually better and more complete results, not only in the assessment processes, but also in decision-making, in tackling the existing in real world uncertainty and possibly in various other human or machine activities. This is, therefore, an interesting subject for further research.

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