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COGNITION HORIZON AND THE THEORY OF HYPER-RANDOM PHENOMENA

Igor Gorban

Abstract: *In the generalized paper, materials of physic-mathematical theory of hyper-random phenomena oriented to description of statistically unstable physical phenomena are presented. Cognition questions of the world are discussed from position of this theory. Different measurement models are researched. The hypothesis of statistical unpredictability and the hypothesis of hyper-random setting up of the world are analyzed. From these hypotheses follow that cognition horizon is restricted by boundaries of unpredictable changes of researched physical phenomenon and of statistical conditions of their observations.*

Keywords: *hyper-random phenomena, setting up of the world, measurement models, cognition.*

ACM Classification Keywords *G.3 Probability and Statistics*

1. Introduction

How our world set up is, how is a human looking for it, what is cognition process? These are the fundamental questions that have been disturbing the humanity for a long time. Probably, it is impossible to find exhaustive answers to these questions. Here we have been encountering with the problem, well known in philosophy that it is impossible to rigorously prove any natural science theory.

There are in the basis of every natural science theory some statements that although well matched with a lot of experimental data however cannot be rigorously proved. These statements are accepted on faith and are used as absolutely reliable source until they will be revised. In philosophy science these statements are known as fundamental abstract objects [Степин, 1999]. The example of such statements is classic lows of Newton's mechanics seemed unshakeable for a long time. However lack of correspondence between results of a number new experiments and classic physical lows discredited assertion that the world can be sufficiently described by these lows and led to forming new lows that became the basis of quantum mechanics and Einstein's theory of relativity.

Mark, mathematics contains unproved elements – axioms and postulates too.

The study base of the world is statements-hypotheses that well consistent with experimental data. They are accepted on faith because are not proved and play rule of axioms. They lie inherently of any theory.

The main demands for the system of basis hypotheses are the consistency and the mutual independence. For the hypotheses of natural sciences it is requested accordance with experimental data too. Hypotheses of natural sciences may be generalized type, for instance the hypothesis that in the base of the universe is random principles and therefore the world is sufficiently described by stochastic models (this standpoint is a prevalent hypothesis now) or more concrete type one, for instance that the world is described by Newton's lows.

Replacement of real objects, relationships, and operations by definite hypotheses (axioms) essentially reduces cognition process and supplies stable base for study of the world, however it creates insuperable (within the bounds of accepted hypotheses) obstacles for penetration to the essence of physical phenomena. There is paradoxical situation: the cognition is impossible without systems of hypotheses but existence of such systems hold back understanding of basis of the universe.

The multitude of research results of any real phenomenon obtained on the base of different systems of axioms can be regarded as a complex of projections on a number of abstract planes. The more systems of hypotheses and consistent theories created on them, more projections and dipper penetration to the base of the world. Therefore the building of new theories that gives possibility to view on the known facts from new points of view supplies the progress of science.

The science is developed by extension manner, if hypotheses system is fixed. For intensive development new alternative hypotheses and theories are requested. Humanity progress is unthinkable without creation of new theories.

Any mathematical theory is an abstract one. It stays such type before it is not used for solving of practical tasks. Correct using of mathematical theory in different areas of physics, technique, social science, and other ones is possible only when experimental data that conform of adequate description of researched phenomena by corresponding mathematical models are existed. For instance, for correct using of probability theory, experimental data that conform of adequate description of physical events, magnitudes, processes, and fields by random (stochastic) models are requested.

It is not possible to create mathematical models those absolutely identical to real physical phenomena. Even if such models exist it is impossible to prove their adequacy because the accuracy of any measurement is finite.

It is possible to estimate the adequacy of the models to real research objects when there are number experimental data. Different methods can be used for this aim. In any case, it is impossible to obtain the absolutely accurate answer. Therefore, only the *adequacy hypothesis* is accepted for the model if the estimate of adequacy is high.

An adequacy hypothesis is a physical hypothesis that open door for correct using of the mathematical theory in practice. The mathematical theory becomes the physic-mathematical theory thanks to this hypothesis.

For instance, the mathematical probability theory based on the mathematical axiom system only after acceptance of addition *hypothesis that real phenomena may be described adequately by stochastic models* becomes the physic-mathematical theory.

Wide application of any mathematical theory points that there is an adequacy hypothesis and also there is the agreement that visual environment (or its part) is created on the principles of this hypothesis. In particular, occurring everywhere using of the probability theory means that the *hypothesis of random* (*stochastic*) *principles of visual environment* is accepted.

The thesis of stochastic character of visual environment was considered as unquestionable one. However a number facts point that this thesis is a fallacy one.

Recent methods and models of probability theory were developed mainly for statistically uniform (for statistical stable) phenomena samples of which are described by no changed distributions. Just statistically stability of physical phenomena was the bench mark for founders of probability theory.

The hypothesis about adequate description of physical phenomena by stochastic models is well matched with experimental data on small temporal, space, or temporal-space observation intervals. However, it is unjust on large intervals.

One from the most essential argument against the hypothesis of stochastic character of visual environment is *absence of global statistical stability* of real physical events, magnitudes, processes, and fields [Горбань, 2007]. There are not absolutely stable phenomena in the real physical world. Exceptions may be only world physical constants such as light velocity, gravitation constant, and so on. Mark, it is impossible to prove their existence by experiment ways because the accuracy of real measures is limited.

Finite accuracy of any physical measurement is the second weighty argument against the hypothesis of stochastic character of the world [Горбань, 2007]. Classic probability theory and mathematical statistics deal with consistent estimates that lead to true values when the sample volume tends to infinity. Errors would be not limited if physical world is adequately described by stochastic models of consistent type. But it is not so.

The searching of effective means of adequate description of real world has produced numbers of new theories. As a rule, they have interdisciplinary character and are touched with mathematics, informatics, physics, philosophy, and other sciences.

The theories created on the paradigms of probability theory [Колмогоров, 1936], fuzzy-technology [Zadeh, Kacprzyk, 1992, Batyrshin, Kacprzyk, Sheremetov, Zadeh, 2007, Бочарников, 2001], neural network [Hagan, Demuth, and Beale, 1996], dynamic chaos [Cronover, 2000, Дыхне, Снарский, Женировский, 2004, Анищенко, Вадивасова, Окрокверцхов, Стрелкова, 2005, Гринченко, Мацыпура, Снарский, 2005, Sharkovsky, Romanenko, 2005], interval data [Шокин, 1981, Алефельд, Херцбергер, 1987, Кузнецов, 1991, Shary, 2002, Kreinovich, Berleant, Ferson, Lodwick, 2005, Ferson, Kreinovichy, Ginzburg, Myers, 2003], and others are related to them. New physic-mathematical theory of the theory of hyper-random phenomena [Горбань, 2007] is in this list too.

The aim of the paper is the discussion of cognition boundaries of the reality in the aspect of the hypothesis of hyper-random setting up of the world advancing in the theory of hyper-random phenomena.

The paper consists of nine sections. Brief description of mathematical and physical bases of the theory of hyper-random phenomena is presented in Section 2. Unformalized, physical, and mathematical models and also processes of knowledge, reasoning, and cognition are submitted in Section 3. Measurement questions of physical quantities are researched in Section 4. The mathematical basis of measurement, the problem of forming of adequate estimates and models are considered hear. Modern approaches used for estimation of measurement accuracy are discussed in Section 5. Different measurement models are described in Section 6. The hypothesis of statistical unpredictability and the hypothesis of hyper-random setting up of the world, that are the base of the theory of hyper-random phenomena are discussed in Section 7. These results give possibility to understand why accuracy of any physical measurements and cognition of the world are limited. The results demonstrating the nonlinear character dependence of measurement accuracy from the volume of data smoothing are presented in Section 8. Last Section 9 is devoted to conclusion.

2. Mathematical and Physical Bases of the Theory of Hyper-random Phenomena

The theory of hyper-random phenomena has two components – mathematical and physical ones. Mathematical part is an advancement of classic probability theory and mathematical statistics. Physical part is based on the hypothesis of statistical unpredictability and the hypothesis of hyper-random setting up of the world. According to the first one, there are unpredictable phenomena in the world that not depend from any events occurred before. According to the second one, it is possible to describe sufficiently the real physical world by hyper-random models.

The basis hypotheses using in the theory of hyper-random phenomena include Kolmogorov axiom system of probability theory and the hypothesis of hyper-random setting up of the world.

The concept of randomness and random phenomenon (event, quantity, process, or field) are understood by different researcher in different ways. Mark, all initial conceptions, used in science, including randomness concept, are accepted by agreement. It was written a lot about this [Тутубалин, 1972, Тутубалин, 1993, Алимов, Кравцов, 1981, Кравцов, 1989].

In the probability theory, Kolmogorov set-theoretic definition of the random event [Колмогоров, 1936] is used usually. Strictly mathematically, a random event is described by the probability space $(\Omega, \Im, P)$, where Ω is a sample space with elements $\omega \in \Omega$, $\Im$ is a sigma algebra of events, P is a probability measure defined on the sigma algebra $\Im$. Random event is defined in such manner even in the International Standard [International standard, 2006].

According to less strict but more illustrative statistical definition (by P. von Mises [Mises, 1964, Гнеденко, 1961]), the probability $P(A)$ of a random event A is a limit of the frequency $p_N(A)$ of its occurrence during experiments in equal conditions when a number of the experiments N tends to infinity: $P(A) = \lim_{N \to \infty} p_N(A)$. With low values of N frequency $p_N(A)$ can vary, but with increasing of N it gradually stabilizes and with $N \to \infty$ it tends to a definite quantity $P(A)$.

It is known algorithmic definition of the randomness proposed by Kolmogorov in the middle of 60th years of past century. It basis on the analyzing of algorithmic complexity of the program, that translates a known sequence to the researched one [Колмогоров, 1987].

It is known other approaches too. On practice, as a criterion of the randomness it is used often by physicists a lot of different empirical, semi-empirical, semi-formalized, and even non-formalized criterions such as a fall down correlation, a continues spectrum, a irreproducibility, a non-repeatable observation, a non-controllability, a unpredictability, and so on [Кравцов, 1989].

In current paper, Kolmogorov set-theoretic definition of the random event is used. Random quantity is a deterministic numerical function defined on a sample space Ω of random events $\omega \in \Omega$. Random function is a deterministic numerical function of independent argument, value of which under fix value of argument is a random quantity.

When one says about a hyper-random phenomenon (as a mathematical object) a set of the condition random phenomena (events, quantities, or functions) depending from the condition $g \in G$ is implied.

Hyper-random phenomena can be described by means of tetrad $(\Omega, \Im, G, P_g)$ [Горбань, 2005, Gorban, 2006, Горбань, 2007] where Ω is a space of elementary events $\omega \in \Omega$, $\Im$ is a sigma algebra of events, G is a set of conditions $g \in G$, P_g is a probabilistic measure of the subset of events depended on a condition g. Thus, the probabilistic measure is defined for all subsets of the events and all possible conditions $g \in G$ while the measure for conditions $g \in G$ remains undefined.

By using less strict approach, conditions g cab be regarded as statistical conditions that associated with definite distributions and a hyper-random event A can be treated as the event which frequency of occurrence $p_N(A)$ does not stabilize with increasing in the number of experiments N and has no limit, when $N \to \infty$.

As probability distribution fully characterizes random phenomenon, as set of condition probability distributions fully characterizes hyper-random phenomenon. For instance, a random quantity X is fully described by any distribution function $F(x)$ (Fig. 1, *a*) and a hyper-random one $X = \{X / g \in G\}$ – by set of conditional distribution functions $F(x/g)$, where g is an element of the set of statistical conditions G.

Expectation, variance, and other moments less fully characterize random quantity. Mark, if one or some moments of random quantity are known (or given), it is implied that there is definite (may be unknown but single) distribution low that fully describes examined random quantity.

Hyper-random quantity is described less fully by upper $F_S(x)$ and lower $F_I(x)$ boundaries of the distribution function (Fig. 1, *b*), by their central and crude moments (expectations of boundaries m_{Sx}, m_{Ix}, variances of boundaries D_{Sx}, D_{Ix}, and so on), by boundaries of moments (expectation boundaries $m_{ix} = \inf_{g \in G} m_{x/g}$, $m_{sx} = \sup_{g \in G} m_{x/g}$, variance boundaries $D_{ix} = \inf_{g \in G} D_{x/g}$, $D_{sx} = \sup_{g \in G} D_{x/g}$, where $m_{x/g}$ and $D_{x/g}$ are accordingly expectation and variance of condition random quantity X / g) and others.

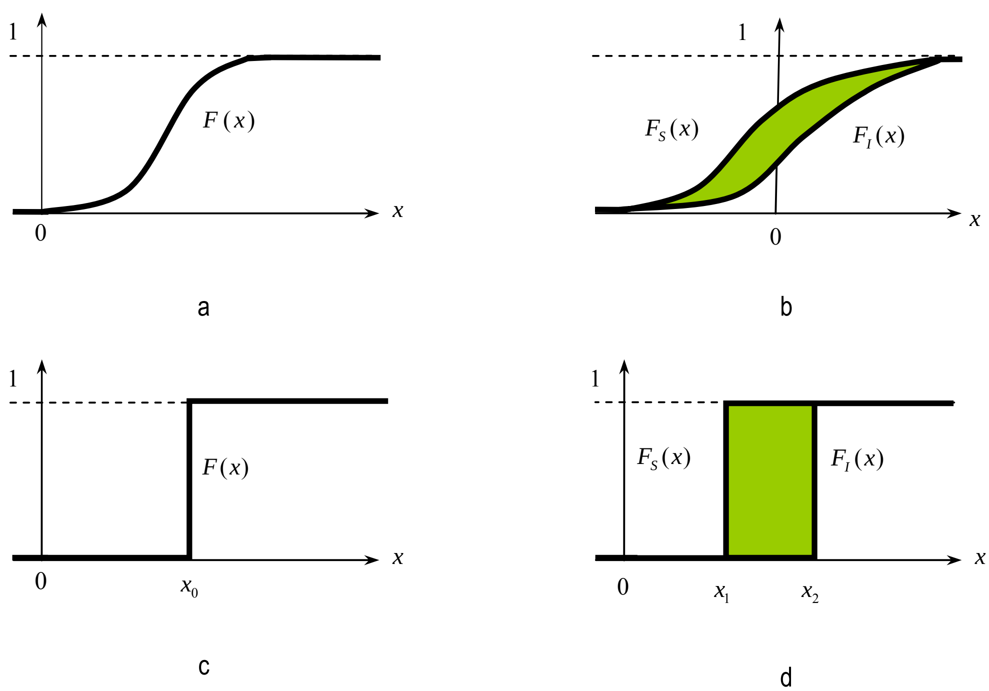


Fig. 1. Presentation of different type quantities by distribution functions.

When any hyper-random quantity is researched, it is implied that there is definite (may be unknown but single) set of condition probability distributions that fully describes of it.

Mark, that a random quantity is a particular case of a hyper-random quantity in which boundaries of the distribution function are coincided: $F_S(x) = F_I(x) = F(x)$.

Determinate quantity x_0 can be regarded in rough as a particular case of a random (or hyper-random) quantity that has in the distribution function a unit step in the point x_0 (Fig. 1, *c*).

Interval quantity [Шокин, 1981, Алефельд, Херцбергер, 1987] characterized by interval boundaries x_1, x_2 can be offered as infinite number of hyper-random quantities with unit step boundaries of the distribution function $F_S(x)$, $F_I(x)$ in the points x_1, x_2 (Fig. 1, *d*).

A special case of the interval quantity $[x_1, x_2]$, when x_1 tends to minus infinity and x_2 – to infinity, is fully ambiguity quantity that can be considered as “chaotic” one not given in to any description.

So, a hyper-random quantity is a generalized concept of determinate and random quantities and all set of hyper-random quantities with definite distribution boundaries is a generalized concept of interval and ambiguity quantities.

All pointed quantities can be interpreted as same type objects with different determinism level. Determinate quantity has the highest determinism level. Determinism in the random quantity is on the level of statistical distribution low, in the hyper-random quantity is on the level of concrete conditional distribution functions, and in the interval quantity is on the level of determinate boundaries of interval. Ambiguity quantity has not any determinism elements.

First papers devoted to hyper-random theory were published in 2005 [Горбань, 2005]. For passed years the theory was developed in many directions [Горбань, 2006, Gorban, 2006, Горбань, 2007, Горбань, 2008, Gorban, 2008]. The classic principles of probability theory and mathematical statistics were used [Колмогоров, 1936, Гнеденко, 1961, Тутубалин, 1972, Королюк, 1985, Боровков, 1986] on the basis of it.

The theory of hyper-random phenomena covers hyper-random events, hyper-random quantities, and hyper-random functions. Results firstly obtained for one dimension case were generalized on multidimensional case too. Number new definitions and conceptions were inputted in particular convergence, continuity, differentiability, and integrability of hyper-random quantities and functions and also stationary, ergodic, and Markov hyper-random processes. Particularities of different hyper-random quantities and functions were researched. Conception of hyper-random sample was inputted, properties of estimates of hyper-random quantities and processes were researched, in particular their convergence.

It was devoted [Горбань, 2007] that in general hyper-random estimates are not consistent ones, i.e. their errors do not follow to zero when the volume of the sample tends to infinity. This circumstance is very important. Hyper-random estimates substantially differ from analogues random ones by this particularity.

To obtain image about physical aspect of the theory let us examine a classic stochastic example, from which the learning of probability theory begins – the “toss-up” coin game. It researched in detail by a lot of mathematics. Pearson made $24,000$ tossing of the coin [Тутубалин, 1972]. The heads was obtained $12,012$ times. The same type experiments were led by Bernoulli, Laplace, and other mathematicians. The task offered not trivial for them.

It is assumed generally that results of the experiments are random and have specific probabilities: the probability of heads is $P_h = 0.5$, and the probability of tails is $P_t = 0.5$.

Is the model described correct? It seems, ex facte, there are no reasons to doubt its adequacy. Even Pearson’s experiment proved this.

However, it is true at first sight. Is it not possible the probabilities P_h, P_t be different? It is easy to show that with some training and having the controlled initial position of the coin one can learn to toss it so the dropout rate of a side will be within the range of some fixed value larger than 0.5, and the dropout frequency of the other one – within the range of the value less than 0.5. Indices can change in either side when conditions of tossing vary.

In this case, the results of the experiments can be treated as a hyper-random event. Thus, hyper-random model taking into account possible changes of probabilities for heads and tails describes real situation more adequately than the random one which assumes fixed values of these probabilities.

3. Cognition of the World

The environment is a complicated system containing numberless interconnected elements – objects. Every object is characterized by a lot of properties determining its peculiarities. Some properties of different objects can be coincided or be closed on definite criterion. Objects with identical or near properties are grouped in our mind in classes of similar objects.

Every object can belong to several classes. Classes can be included in other classes. Properties that proper to similar objects are regarded as regularities or laws. Regularities and laws are objects too. As other objects they can form classes. Regularities and laws define specific peculiarities of connections of objects of the class with other objects of the same class and with objects of others classes.

Classes, for instance, are the multitude of physical objects that obey to lows of classic mechanics, all known lows of classic mechanics, objects sizes of which are less or larger to definite magnitude, plants or animals of definite type, people of definite nationality, confession, race or age, citizen of the country, and so on.

Systematization (forming the system of interconnected object classes) and classification (distribution of objects to classes) are the main actions in cognition of the world. They include detection new objects, examine and description of their properties, replenishment of existed classes by new objects, creation of new classes, and removal of outdated classes from the system.

As a result of systematization and classification, unformalized models that give integral presentation about similar classes are formed in the mind.

Unformalized models are not identical to examined objects. They carry information about multitude of similar objects included to class and are theirs averaged image data.

Everybody tries to attribute every new object, as a rule subconsciously, to different unformalized models. The models to that the object mostly accords are associated with the object and contrary the object is associated with these models. As a result, a multitude unformalized models are formed. This multitude is interpreted by person as integrated image of the object. A set of images according to different objects creates subjective image of the person about surrounded world.

The world is continuously changed; objects and their models are changed too. Changing of the models is connected not only by changing of the objects but revision of classification criteria. When a criterion is chosen, essential role plays acquired experience, environment, and many others factors.

Persons are differed each other and conditions of their life are differed too. Therefore unformalized models of the objects, images, and world conceptions are different for different persons.

Real objects are perceived by a human by sense organs as estimates that can be regarded as objects too.

Estimate depends from a lot of subjective and objective factors (conditions). The estimate is differed from the real object as by imperfection of our sense organs and observation means as different noise effects. When conditions are changed (for instance, sight become weaker, more or less perfect observation means are used, noise characteristics are changed) the estimate is changed too.

A model and an estimate are different conceptions. If the model of the object is the generalized image about the multitude of similar objects, the estimate is more or less distorted image of the single object.

On the base of estimates the average estimate and estimate model are formed.

A multitude of different estimates corresponding to single object or some objects from definite object class creates in human mind the unformalized model of the estimate. This model as an object model is continuously changed. Changes are called by changing of objects, estimates, and classification criteria of objects and estimates. Unformalized models of estimates are the basis for forming of unformalized models of objects.

Unformalized models of estimates and unformalized models of objects are different category however as a rule they are identified in human mind.

Unformalized models of objects and estimates can be formalized by physical models, taking into account the most essential their peculiarities, and be described by mathematical models.

A set of physical models can be built for every multitude of real objects and every multitude of estimates. Every physical model can be described by the multitude of different mathematical models.

Part objects that belong to the examined object's class cannot be described by physical model on the phase of formalization. Together, objects not belonged to the examined class can be included in the physical model. As a rule, when mathematical model is built, all objects of the examined class that belong to the physical model are described by mathematical model.

Similar objects of any class, their estimates, examined object x, its estimate x^*, models of the object (unformalized N_x, physical P_x, and mathematical M_x) and also the models of estimates (unformalized N_{x^*}, physical P_{x^*}, and mathematical M_{x^*}) are described schematically in Fig. 2.

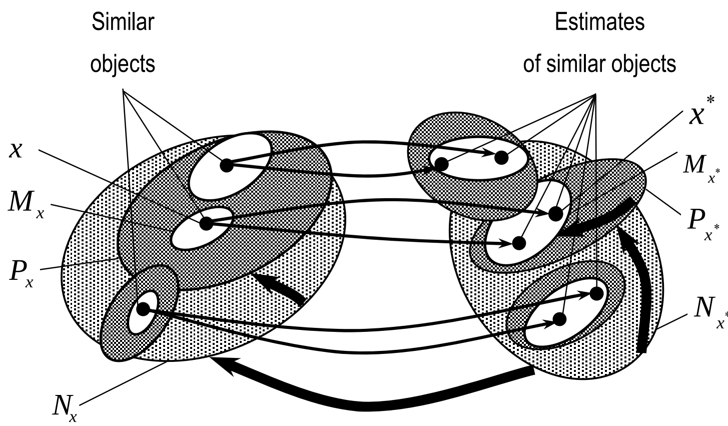


Fig. 2. A scheme of the models and the estimates.

Knowledge is a multitude data in any area. A lot of knowledge about the world has been accumulated over millenniums of civilization development. Inflow tempo of new information increases continuously. It becomes more and more difficult to orientate in information flow. Well-tested mechanisms of data systematization, classification, and generalization help to solve the problem.

Human knowledge is not a number of odd data. It is a system of systematized, classified, and generalized data that is a system of models.

Human possibilities in perception and processing of information are limited. The cognition mechanism based on the model system defends human organism from information overload.

Human knowledge is formed on the basis of the knowledge of separate persons by their systematization, classification, and generalization.

Person knowledge and human knowledge are consisted from formalized and nonformalized models. As a rule, models in exact science are formalized and in humanities are nonformalized.

Person image about the world, his world-view is a set of models forming knowledge and person's estimates of these models on a multitude criteria for instance risk, reliability, significance, novelty, accordance to definite regulations, and so on.

Mark that estimates of models and models of estimates are absolutely different categories. If criteria are depended, estimates on different criteria are linked. In spite of person's individuality, knowledge and estimates of different humans in definite object area may be similar. Persons with the same views, joint interests, the same religion, belonged to the same ethnos, and obtained the same education are humans with similar models and analogues person's estimates of models on the multitude of different criteria. Subjective estimates of models are basis for collective estimates.

Models and estimates, as subjective as collective ones, are relatively stable objects that are slowly changed in time under the influence of external factors. Therefore the world-view may be regarded as the system of stable models with stable estimates.

Models can have different level of generalization. On the basis of low level models, creation of more high level derivative models is possible.

Models may be divided to two classes: to models of structural elements and to links among them. The last are the models of real lows of the nature.

Synthesis of new models of structural elements, finding new links, forming new estimates, and their revision are the essence of the thought.

Curiously enough in the first view, creation of derivative models of structural elements, finding of new links, forming of new estimates, and their revision are possible owing to stability of models of structural elements, links among them, and their estimates.

Thought process depends from the degree of stability of initial knowledge: the more stability the more levels of derivative models of structural elements, links, and estimates may be formed.

As show a lot of biology researches, a human substantially inferiors to many animals in possibilities of perception of information, processing of information, random-access storage (i.e. in possibility to form initial models), however owing to definite “conservatism” (thought inertness), he considerably exceeds animals in finding of links among models and in synthesizing of derivative models.

The models of structural elements, the links, and the estimates are changed under the influence of different factors with the lapse of time. New models, links, and criteria of estimation are created, specifications of elements, exclusion of outdated and unclaimed ones, changing of them to new elements are occurred, and also forming and revision of estimates are taken place.

Cognition is a process of knowledge acquisition and comprehension of the lows of the objective world. Cognition by a person and by the humanity may be divided on two phases. The first phase is forming of initial knowledge (initial set of the models) and estimates. The second one is actualization (revision, specification) of existed knowledge and estimates.

In the first phase, nonformalized models play a special role; in subsequent phase, as nonformalized as formalized ones are important.

Both phases are teaching. Person's teaching may be without or with outside help. Ways for knowledge transmission are different: by communication of the persons or by indirect ways with using auxiliary means such as books, computers, video techniques, and so on. Described model of forming knowledge is presented schematically in Fig. 3.

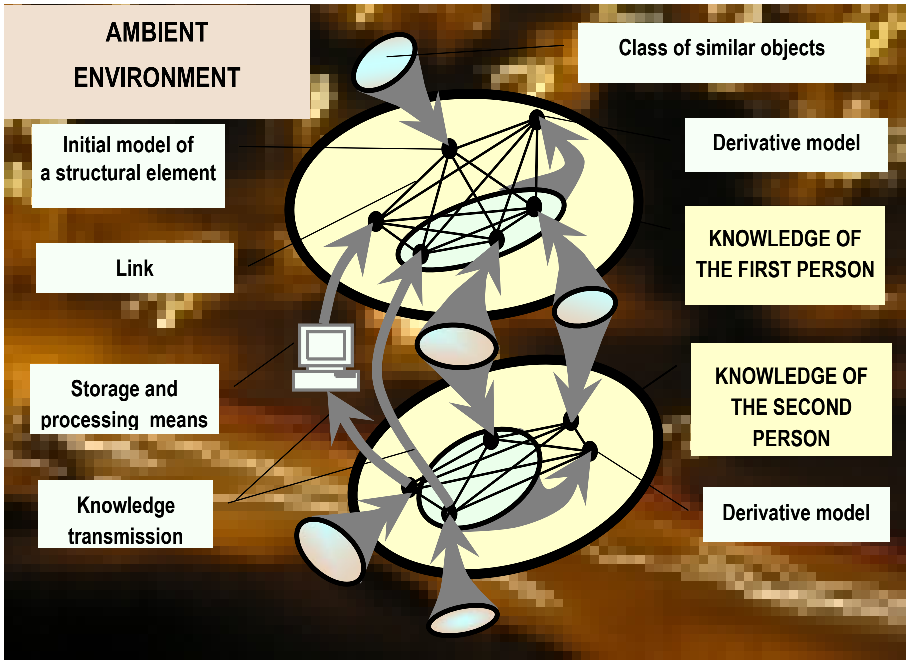


Fig. 3. A scheme of the process of knowledge forming.

Cognition is a complex compound process demanding the forming of the estimates, creating of the models, comparison, confrontation, systematization, classification, detection, and many other operations. In the basis of them is measurement. Cognition facilities are defined by measurement precision. Let us stop on this question at grate length.

4. Measurement Principles

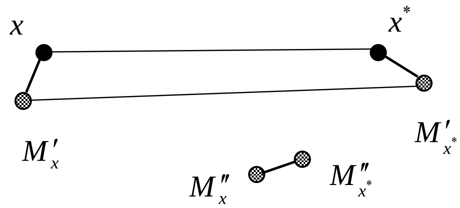
All nonformalized, physical, and mathematical models of objects and estimates are objects. To characterize numerically the closeness of the objects the metric space is demanded. Such space is a multitude S for any elements of witch x, y there is a real function $\mu(x, y)$ that is the metrics or distance that satisfy to following postulates: $\mu(x, y)=0$ if and only if $x = y$; $\mu(x, y) \leq \mu(z, x)+\mu(z, y)$ (inequality of triangle), where x, y, z are any elements of the multitude S . The metrics is a nonnegative quantity and $\mu(x, y) = \mu(y, x)$.

It is known that there are some multitudes for which it is impossible to create a metric space. For instance, it is impossible to build any metric space for the multitude of real functions defined in the finite range. Therefore it is impossible to create a metric space that includes the model of the real object and the models of all its possible estimates. If to restrict the class of considered functions, for instance, to continuous real ones, it is possible to create metric space. In this case, it is possible to define the distance for all mathematical objects described by such functions.

Different metrics generates on a definite multitude S the different metric spaces. The variation distance row according to elements $x, y, z, \ldots$ depends from the metrics. So in two different metric spaces created on the multitude including the model of real object and the models of its estimates, the nearest to the model of real object may be different models of estimates.

It may be case when for a whole class of metric spaces given on the definite multitude the same estimate model is a nearest one for the model of real object. Reference quantity (conventional unit) is settled by the metrics. A distance is compared with this unit. Mark that Euclidean metrics usually is used in the metrology.

If the model of the object M'_x is near to the object x, the model of the estimate M'_{x^*} is near to the estimate x^* ($\mu(x, M'_x) \sim 0$, $\mu(x^*, M'_{x^*}) \sim 0$), or the model of the estimate M''_{x^*} is near to the model of the object M''_x ($\mu(M''_x, M''_{x^*}) \sim 0$) it is not ensure that the estimate x^* and its models M'_{x^*}, M''_{x^*} are near to the object x (Fig. 4, *a*). The estimate x^*, the model of the estimate M_{x^*}, and the model of the object M_x may be near to each other however far from the object x (Fig. 4, *b*).



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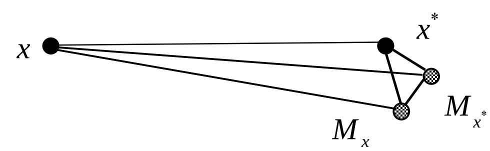

b

Fig.4. Mutual disposition of the object, the estimate, and their models.

Often on the practice, for confirmation of the theory the closeness of theoretical results to experimental data are inspected. Mark, that low difference of the results often regarded as irrefutable reason that the theory is correct, in real is not such one. The experimental result is the estimate and the theoretical result is the model of another estimate. The closeness of these results indicates that the results close to each other but not points that they close to the researched object. The adequacy question of the theoretical model remains open before the closeness of the experimental result to the researched object is not proved.

True (undistorted) information about real object x is inaccessible. It is impossible to know how close are the estimate x^*, its model M_{x^*}, and the object's model M_x to the object x.

Therefore the building task of adequate estimates and models has not exact solution. In future, under adequate estimates and models it will be implied ones that are in the round of the corresponding examined objects.

Coincidence of the estimate, the estimate model, or the object model with the real object practically is impossible. There are a lot of reasons for this. The most essential ones are:

- taking into consideration under forming of the physical models not all factors defining the state of the real object and its estimate;
- forming of the object's model on the base of the estimate's model;
- unmatched parameter changes of the real object and its estimate;
- "delaying" of physical models from the current state of the object and the estimate;
- "delaying" of mathematical models from physical models;
- statistical unpredictability of the real object and its estimate;
- influence of different noises that leads to distortion of the estimate, the estimate model, and the object model;

– statistical unpredictability of noises;
– imperfection of physical and mathematical models, and others.

The distance between the estimate and the object (physical quantity, process, or field) is the measurement error. The error is caused by inadequate perception of the object and by noises. Inadequate perception may be called as by objective as subjective reasons. Noises are connected as a rule with objective factors.

Different character of reasons that calls difference of the estimate and the real object requires different approaches for taking into consideration and compensation of error components. As a rule, the same strategy is used: data accumulation and their averaging.

Compensation of inadequate perception of the object called by objective reasons is based on forming of a number estimates obtained by different ways, compensation of inadequate perception called by subjective reasons is based on multiple estimation of the object by different persons, and compensation of noise influence in statistically stable conditions – on repeated estimates obtained in these statistical conditions.

Results of multiple estimation of the object by different ways and persons are used for obtaining of the average estimate. This estimate although as a rule is nearer to the real object than the most part of initial estimates but not coincide with the real object. This is so not only because the data volume is limited and the average mean is not optimal. The main reason is that it is impossible to watch changes of characteristics of the object and noises. As a result, measurement precision always is limited.

Mark, measurement accuracy is a qualitative category quantitatively characterized by error or uncertainty of measurement.

5. Bias Conception and Uncertainty Conception

In the metrology two approaches are used to characterize measurement precision. One of them is based on bias conception and another one (that has reputation of more progressive) – on uncertainty conception.

In bias conception, systematic and random errors are regarded. Systematic error is that one which remains fix or changes on definite low when multiple measurements are led. Random error is that one which changes by a random manner when multiple measurements are led.

Random error usually explains by time or space random changes of quantities, that influent on the result of measurement, and systematic error – by deviation of parameters or measurement conditions from ideal ones.

Random error may be reduced by statistic processing of number results of measurements and systematic error – by taking into account of known dependences of measurement results from parameters influencing on the results.

If the systemic error not changes from measure to measure (this fact is accepted, as a rule by default) the systematic error coincides with expectation of the cumulative error. In this case, expectation of the random error equals to zero.

The error of the estimate Θ^* of the measured quantity θ are usually characterized or by the systematic error ε_0 (expectation of the error) and the standard deviation σ_{θ^*} of the estimate Θ^* or by the confidence interval $I_\gamma(p) = [\Theta^* - \varepsilon_0 - \varepsilon, \Theta^* - \varepsilon_0 + \varepsilon\ ]$ according to the definite confidence probability $\gamma = P(\left|\theta^* - \varepsilon_0 - \theta\right| \le \varepsilon)$ that the absolute deviation of the random quantity $\Theta^* - \varepsilon_0$ from the measured quantity θ is not more than the definite volume ε.

In some cases, the systematic error can be compensated partly by special measurement methods that give possibility to reduce it without error detection. It is known many such methods, in particular the substitution method, the error compensation on sign method, opposition method, symmetric surveys one and others.

The random error can be reduced by multiple estimation of the quantity and averaging of the obtained data.

In uncertainty conception, two types evaluations of uncertainty (A and B) are regarded. Type A evaluation of uncertainty is a method of evaluation by the statistical analysis of series of observations and type B evaluation of uncertainty is a method of evaluation by means of other manner [Guide, 1993].

Uncertainty of the measured quantity θ is characterized by type A standard uncertainty $u_{\mathrm{A}\theta}$, by type B standard uncertainty $u_{\mathrm{B}\theta}$, combined standard uncertainty $u_{\theta} = \sqrt{u_{\mathrm{A}\theta}^2 + u_{\mathrm{B}\theta}^2}$, and expanded uncertainty $U_{\theta} = k u_{\theta}$ (were k is a coverage factor) that in case of absence of $u_{\mathrm{B}\theta}$ component is interpreted as uncertainty according to confidence probability γ [Guide, 1993].

Dividing of errors on random and systematic is caused by nature of their formation and manifestation in the process of measurement, and dividing of uncertainty on A and B types – by evaluation methods.

Underline, although these approaches have essential differences, both concepts are based on the proposition that errors can be called only by deterministic and random factors. Other types of factors, in particular of hyper-random type are not taken into account. Seemingly this orthodox position requests the revision.

Let us consider the example related to the metrology – the precise measurement of diameter of the cylinder of circular section [Горбань, 2007]. The completely trivial problem appears quite complicated under in-depth analysis. To make a detail of absolutely circular section is impossible. Its section will always differ from ideal circle: firstly, because of ellipsoidal or any other deviation from ideal circular form, and, secondly, due to roughness of the surface. It should be also born in mind that different sections by the cylinder axis are differed. Therefore, the true size of the detail, even without considering temperature and a number of other factors, which can be ignored further in order to simplify the speculations, is different in different measurements.

In this case, because of complex shape of the detail section the concept of the diameter is not acceptable. Considering this, the problem should be defined as the measurement task of average size of sections.

The physical model of the measurable value should to consider the deviation from the ideal circular shape, roughness, and difference in sections along the axis. For mathematical description of physical model, in principle, both common random and hyper-random mathematical models can be used.

The random model is based on an assumption that probabilistic characteristics of the measurement results are constant. In reality, this is not true. Within small local areas they can be approximately constant, but in general they can considerably depend on a direction along which the measurement is taken, and on the section under consideration. Therefore, hyper-random model, considering the variability of distribution functions, better describes the measurable value than the random model.

Any measurements are carried out under impacts of various obstructive factors (obstacles). In the examined problem, such is the clogging of surface of the detail. Dust and dirt on the surface accumulates unevenly. Within small local areas clogging has random type, but in whole, because of difference of distribution laws for different areas, it is of a hyper-random type.

There are no absolutely precise measurement instruments in the world. Neither a trammel, nor a micrometer or any other measurement instrument can measure the section dimensions with infinitely high precision. Reasons are different: roughness of the detail surface, various instrumental errors etc. The unifying feature of them is that they are of hyper-random type. Therefore, for mathematical modeling of a measurement instrument the hyper-random model is preferable.

Hence, it follows that a generalized physical model, considering in whole real features of measurable value, obstacles and measurement instruments, is described more adequately by a hyper-random mathematical model than the random one.

Mark, the task of measurement of physical quantity in conditions of statistical instability is not knew [Тутубалин, 1972]. A lot of mathematicians and physicians discussed the problem. In the middle of former age number (Kolmogorov, Pearson's chi-square, omega-square, and others) testing nonparametric statistic hypotheses methods that give possibility to evaluate statistical homogeneity of the sample were developed.

Examine now the classic measurement model used in metrology and other ones [Горбань, 2006, Gorban, 2006, Горбань, 2007, Gorban, 2008].

6. Measurement Models

In the traditional classic notion it is supposed that the measured physical quantity θ is not changed for measurement time and the result of measurement (estimate Θ^*) is changed from measure to measure on a random low, so there is a statistical steadiness of the estimate. Taking into account that the measured quantity is considered as a deterministic one and the result of the measurement is a random one (Fig. 5, *a*), this measurement model may be called as a deterministic – random type.

A measured quantity can be a random type too (Fig. 5, *b*). According measurement model may be called as a random – random type.

When a measured quantity has a deterministic character and its estimate has a hyper-random character (Fig. 5, *c*) we obtain a deterministic – hyper-random model of the measurement. It may be other types of the models. A most usual model is a hyper-random – hyper-random one. In this model a measured quantity and its estimate are variables of hyper-random types (Fig. 5, *d*).

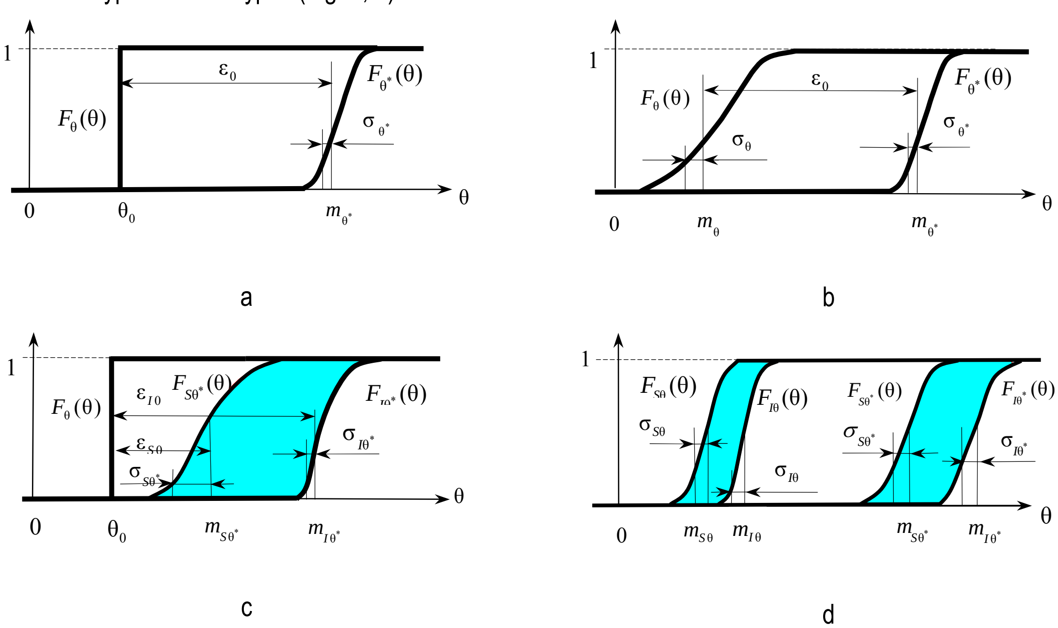

Fig. 5. Different measurement models: deterministic – random (a), random – random (b), deterministic – hyper-random (c), and hyper-random – hyper-random (d) models.

Parameters and characteristics describing measurement accuracy are different for different measurement models. For a deterministic – random model (Fig. 5, *a*) such ones are the distribution function $F_{\theta^*}(\theta)$ of the estimate, the expectation m_{θ^*} of the estimate, its bias $\varepsilon_0 = m_{\theta^*} - \theta_0$, the standard deviation σ_{θ^*} of the estimate, and other ones.

For a random – random model (Fig. 5, *b*) they are the distribution functions $F_\theta(\theta)$, $F_{\theta^*}(\theta)$ of the measured quantity and the estimate accordingly, the expectations m_θ, m_{θ^*} of the measured quantity and the estimate correspondently, the bias $\varepsilon_0 = m_{\theta^*} - m_\theta$ of the estimate, the standard deviations σ_θ, σ_{θ^*} of the measured quantity and the estimate, and others.

For a deterministic – hyper-random model (Fig. 5, *c*) they are the upper $F_{S\theta^*}(\theta)$ and lower $F_I(x)$ boundaries of the distribution function of the estimate Θ^*, the expectations $m_{S\theta^*}$, $m_{I\theta^*}$ of the boundaries of the estimate, the biases $\varepsilon_{S0} = m_{S\theta^*} - \theta_0$, $\varepsilon_{I0} = m_{I\theta^*} - \theta_0$ of the boundaries of the estimate, the standard deviations $\sigma_{S\theta^*}$, $\sigma_{I\theta^*}$ of the boundaries of the estimate, and others.

For a hyper-random – hyper-random model (Fig. 5, *d*) they are the upper $F_{S\theta}(\theta)$ and lower $F_{I\theta}(\theta)$ boundaries of the distribution function of the measured quantity, the upper $F_{S\theta^*}(\theta)$ and lower $F_{I\theta^*}(x)$ boundaries of the distribution function of the estimate Θ^*, the expectations $m_{S\theta}$, $m_{I\theta}$ of the boundaries of the measured quantity, the expectations $m_{S\theta^*}$, $m_{I\theta^*}$ of the boundaries of the estimate, the standard deviations $\sigma_{S\theta}$, $\sigma_{I\theta}$ of the boundaries of the measured quantity, the standard deviations $\sigma_{S\theta^*}$, $\sigma_{I\theta^*}$ of the boundaries of the estimate, and others.

7. Hypothesis of Statistical Unpredictability and Hyper-random Hypothesis of World Organization

Estimates according to different measurement models have different properties. A estimate of expectation of a random quantity obtained by calculating of sample mean is consistent estimate. The variance of this estimate is reduced proportionally to sample volume. Therefore in case of a deterministic – random model, a random measurement error of any constant magnitude can be theoretically reduced to zero when the sample volume tends to infinity.

On practice, possibilities of reducing of the error are limited by the longitudes of intervals where the measured magnitude practically is not changed and statistical conditions of its observation are remained invariable.

Since these longitudes of intervals are always finite quantities the real estimates are not consistent ones. Therefore it is impossible to obtain the unlimited accuracy, even under unlimited data volume. The accuracy is defined by the longitudes of space-time intervals of stability of the researched object and statistical conditions of its observation. With approaching to the highest measurement accuracy the effectiveness of averaging reduces. This shows up in slowdown of the reducing rate of the measurement variance, when the sample volume rises.

Maximum number of the running data of the sample for that the deterministic – random measurement model stays the adequate one can be estimated by known estimation methods for sample homogeneity [Королюк, 1985, Леман, 1979, Горбань, 2003].

When there is a large sample volume the inadequate description of real estimates by random models leads to conclusions not conforming to experimental data. This becomes especially clear when high accuracy measures are led.

In every concrete case, physicists give different explanations why it is impossible to achieve of the extremely high accuracy (for instance, by Brownian motion of molecules). These explanations as a rule are correct. However, great number of them and their variety look from the side as subconscious desire to save classic random model of world organization.

It seems more natural *to accept that in the real world there are statistically unpredictable phenomena* limiting the accuracy of measurement and the possibility of cognition.

Statistical unpredictable hypothesis may be regarded as the physical model of one of fundamental lows of the nature. Its mathematical model is the *hyper-random hypothesis of world organization* assumed that *any physical phenomena can be described adequately by hyper-random models.*

According to hyper-random hypothesis of world organization *all real physical quantities, processes, and fields* (may be excepting only small number of world physical constants) *have hyper-random character* [Горбань, 2006, Горбань, 2007, Gorban, 2008].

This hypothesis concerns to different real phenomena including quantities, functions, and fields that are measured, current noises, and measurement errors. Every estimate is forming as an estimation result of the mixture of the measured quantity, process, or field and the noises prevented for observation.

If noises are of hyper-random type, even in case, when the measured quantity is a constant (world constant) the result of measurement is of hyper-random type.

Hyper-random character of the estimate is shown up firstly in unpredictable drift of the shift. Since the drift is changed in unpredictable manner it is impossible to compensate it.

So it is clear why all estimates of real quantities, processes, and fields are not consistent and potential accuracy of any measurements is limited. The boundary of accuracy defends not only the number of measurements and their random dispersion but mainly unsteady character of probability characteristics of measured quantity and noises.

In fact, hyper-random hypothesis recognizes existence of unpredictable physical phenomena that defines the strategy of world development.

Philosophy idea, based on the existence of unpredictable phenomena, suggested and discussed by many philosophers. But it was not developed to the level of formalized mathematical models that give possibility to obtain strict logical conclusions. The novelty and specific of hyper-random theory consist mainly in that it formalizes this idea and proposes mathematical apparatus for solving different practical tasks.

Hyper-random estimates are not consistent ones while parameters used to describe these estimates are of consistent type. When a sample volume tends to infinity the value of hyper-random estimate Θ^* unpredictably changes in the range $[m_{S\theta^*}, m_{I\theta^*}]$. In many cases, hyper-random estimates due to their unpredictability are more prefer for description of real objects than random ones.

To illustrate specific particularities of developing approaches look at concrete task.

8. Example

Let in uncertainty statistical conditions the measured quantity, the estimate, and the sample are adequately described correspondently by hyper-random quantities Θ, Θ^*, $\vec{X}$. The random sample according to statistical condition $g \in G$ is presented by additive mixture of random quantity Θ / g and random homogenous noise described by a vector $\vec{V} / g$ the components of which are independent and have expectations $m_{v/g}$ and variance $\sigma^2_{v/g}$. The noise does not depend from measured quantity and its variance is in the range of [σ^2_{iv} , σ^2_{sv}]. Statistical condition changes so slowly that sample forming conditions may be accepted as practically invariable ones.

It is necessary to estimate the measured quantity θ/g in uncertainty condition $g \in G$ and the measurement accuracy.

If we have N running samples $x_n / \theta, g$ it is possible to form for uncertainty condition $g \in G$ the estimate $\theta^* / \theta, g = \frac{1}{N} \sum_{n=1}^{N} x_n / \theta, g$.

The middle error squared is $\Delta^2_{z/g} = m^2_{v/g} + \frac{\sigma^2_{v/g}}{N}$. This quantity can be estimated by the following inequality:

$$|\varepsilon|^2_i + \frac{\sigma^2_{iv}}{N} < \Delta^2_{z/g} < |\varepsilon|^2_s + \frac{\sigma^2_{sv}}{N} \tag{1}$$

were $|\varepsilon|^2_i = \inf_{g \in G} m^2_{v/g}$, $|\varepsilon|^2_s = \sup_{g \in G} m^2_{v/g}$ are squares of low and upper boundaries of the module of the estimate shift.

When $N \to \infty$, expression (1) becomes the following one: $|\varepsilon|^2_i < \Delta^2_{z/g} < |\varepsilon|^2_s$.

Clear, it is reasonable to increase the sample volume before this gives perceptible rising of accuracy. As it follows from expression (1) for hyper-random estimates there is a critical sample volume N_0 from that rising of the volume of processing data is not reasonable. In this sense hyper-random estimates are similar to interval estimates [Шокин, 1981].

By using right part of inequality (1) the critical sample volume N_0 can be estimated in the following manner: $N_0 > \frac{10\sigma^2_{sv}}{|\varepsilon|^2_s}$. It follows from this inequality that with decreasing of upper boundary of the module of the estimate shift $|\varepsilon|_s$ and increasing of upper noise variance σ^2_{sv} the critical sample volume rises. If upper boundary of the module of the shift is comparable with upper boundary of noise standard deviation the critical sample volume N_0 is in the region of dozen samples.

Described approaches may be useful for modeling not only physical quantities but also physical processes and fields. They give possibility to find and explain facts unnoticed or inexplained on the ground of traditional approaches.

9. Conclusion

Cognition possibilities are limited. This is called not only by the limited human possibilities in perception and processing information. Main cause consists in unpredictable phenomena occurred in the world.

A lot of data point that nowhere there is absolute statistic stability. In ambient space all objects and processes are changed unpredictably, sooner or later. Researches show that statistical stability of physical phenomena on that founders of classic probability theory and mathematical statistics are oriented can have in real world only local character. In the best case it occurs on small temporal, space, or temporal-space intervals of observation.

The absence of global statistical stability of physical phenomena does not permit to obtain consistent estimates the errors of which tend to zero when the sample volume tends to infinite. This gives rise to essential doubt on adequacy of universally recognize hypothesis that the real world is organized on random principles.

The hypothesis of statistic unpredictability of phenomena that is the physical model of one of the fundamental lows of nature and the hypothesis of hyper-random world organization that is the mathematical model of this physical model help to explain from the single position a lot of key questions of cognition, in particular why

– it is impossible to obtain infinity high accuracy and to create absolutely adequate mathematical model for any real physical object;

– all real estimates and created on them estimate and object models are not consistent ones (when the data volume is rising their errors are not tend to zero);

– it is impossible to prove by any experiments that even the best perfect physical theory absolutely exactly describes nature lows;

– it is impossible by any real measurement means to confirm or refute the existence of world constants;

– it is impossible to make absolutely exact prediction on the basis of the current and former data.

This list can be continued.

From the hypothesis of statistic unpredictability and the hypothesis of hyper-random world organization follow that cognition horizon is defined by the range of unpredictable changing of physical phenomena and conditions of their observation.

* * *

All hypotheses and theories have limited age. It is impossible to foresee exactly if they will be claimed in a future and in what degree will be claimed. True importance of scientific results is covered in a process of time tests. It would be wanted to hope that the hypothesis of statistic unpredictability, the hyper-random hypothesis, and the theory of hyper-random phenomena will be useful for cognition of our world.

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Authors' Information

Igor Gorban *– Principal scientist of the Institute of Mathematical Machines and Systems Problem, National Academy of Sciences of Ukraine, Glushkov ave., 42, Kiev, 03187, Ukraine; e-mail: igor.gorban@yahoo.com*

Major Fields of Scientific Research: Probability theory and mathematical statistics, Theory of hyper-random phenomena, Space-time signal processing in complicated dynamic conditions, Fast multichannel space-time signal processing