APPLICATION OF SOME CYBERNETIC MODELS IN BUILDING INDIVIDUAL EDUCATIONAL TRAJECTORY

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Abstract. *The individual educational trajectory (IET) is a medium-term didactic complex that provides optimal opportunities for developing the creative potential of the learner by taking into account his/her personality. It includes: 1) developing an individual informational environment; 2) tuning the didactical resources; 3) personalization of the interim goals; 4) planning personal learning and research activities; 5) considering the self-organization. In the paper we submit a model of IET called DMT. In contrast to the cognition trajectory developed by Kolyagin and Ganchev the DMT has no linear structure. The IET in our model consists of sections due to the interim goals and each section of it could be perceived as a beam whose spectrum consists of the listed above 5 components. So the architecture of DMT could be associated with the one of the skyscraper Taipei 101 (the abbreviation DMT stands for Didactic Model Taipei). The educational process in any section in the IET could be conducted in different manner that allows different didactic approaches to be applied with respect to the individualities of the learner. The particular didactic approaches we had applied in a case study held in 2011/2012 academic year are strongly influenced by some cybernetic ones in the Consequence Driven Systems (CDS) theory.*

Keywords: *didactical models, individual educational trajectory, elliptic arbelos, Socratic style teaching.*

ACM Classification Keywords: *D.4.8 Performance (Modeling and prediction), I.6.5 Model Development (Modeling methodologies), J.1 ADMINISTRATIVE DATA PROCESSING (Education)*

0. Introduction

The cybernetic models in education are in use as long as the cybernetic idea appears. Among the earliest ones is the model of Hodge [Hodge, 1970] which architecture is shown at figure 1. The model represents in a very simple and clear way the general stream of the educational process from the organizational perspective. The mystery of learning remains out of sight and the process of learning is affected indirectly.

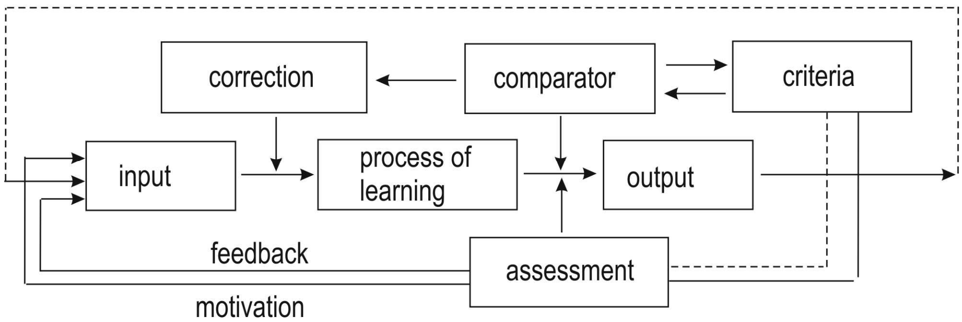

Figure 1. The architecture of the model proposed by Hodge

Quite more sophisticated cybernetic model is the one submitted by Garrison and Magoon [Garrison and Magoon, 1972]. It consists of five blocks each of them representing the mechanism of one particular cognitive process. The interaction of the blocks gives the big picture of the learning process. However, the model is too complicated to be put in use in a regular pedagogical practice. Below we are going to present a model which complexity is somewhere in-between the ones of the two just pointed marginal models. Key role in it play a system of didactic schema that are analogs of cybernetic ones. Our interest in cybernetic models was provoked after attending a lecture of N. Ackovska about taxonomy of the learning agent [Ackovska, 2010].

1. A didactical heritage from modern perspective

The individual approach in education is considered mainly as contrapuntal to the didactical technology. However, some general technological rules could be put in the fundament of any individual teaching-learning process and the first systematic attempt in this direction (as far as we know) is described more than two millennia ago in the Plato's *Dialogues*. In the dialogue called *Meno* [Plato, 4th century BC] we can observe a didactic approach that becomes classics in teaching – the Socratic Method (Fig. 2).

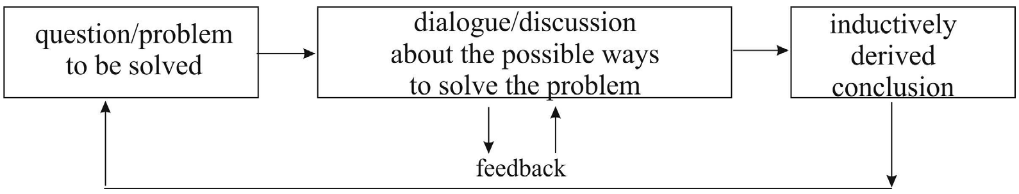

Figure 2. The architecture of the Socratic structure as given by Andreev [Andreev, 1966]

It is emblematic that Socrates illustrated his method with a mathematical example. In modern times this method is developed by various prominent mathematicians and educators like Freudenthal (explicitly [Freudenthal, 1982]) or Polya (implicitly [Polya, 1961]). The genuine Socratic Method of stating questions to rethink the initial problem is too limited for the modern didactics. However, we adopt the style of inductive questioning that leads the student to build his/her knowledge through small steps, to increase understanding through inquiry. Further we refer to such type of teaching as the *Socratic style*. Nowadays close to this style in Europe is so called *inquiry based science education* (for mathematics *problem-based approach*) [Rocard et al, 2006]. The inquiry based education was declared to be potentially effective as general classroom practice. Despite the enormous resources that European educational structures spend to implement the ideas of inquiry based education we are not known convincing evidences for positive breakthrough in mass mathematics education. On the contrary, our belief is that the Socratic style is more effective when it is applied to advanced students combined with the individual approach. The first experimental work that we have done witness such thesis [Lazarov, 2012].

2. The cognition trajectory in teaching mathematics

The concept of *cognition trajectory* is elaborated by Ganchev [Ganchev, 1996] to describe the process of education as a manageable object. The cognition trajectory is think to be an ideal educational process illustrated with a curve that connect the initial cognition status of a learner T_0 with the educational goal G for a (mead term) period. The learner is supposed to 'move along' the cognition trajectory learning the material in full scale (the desired case); if not then 'the learner declines the cognition trajectory' (the case that is much closer to the reality). Two examples of such discrepancy between the desired educational process and the real situation illustrated with cognition trajectory are shown on figure 3.

1) The teacher organizes education supposing the student's knowledge is in a neighborhood of T_1 but the student's knowledge is still in a neighborhood of T_0 (the left picture).

2) The teacher organizes education supposing the student's knowledge has been reached the status T_1 but the student's knowledge went into another direction (in a neighborhood of T') from the beginning T_0 (the right picture).

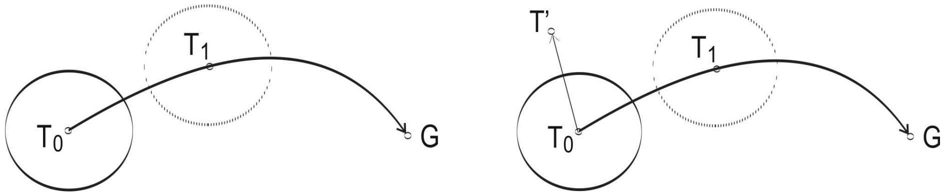

Figure 3. Two cases of discrepancy between desired learning process and reality as given by Ganchev [ibid]

Teacher's intervention is relevant and effective only in the case when student's knowledge is coherent with the cognition trajectory. Ganchev calls such education *properly directed* and lists a system of six main steps which should be performed during the math lessons to ensure proper direction of teaching-learning process. The general idea of the system refers to the Vygotsky's findings about *zones of actual and proximal development* [Vygotsky, 1978] and follows the rule: any educational activity should not leave the student's *zone of proximal development*.

3. The individual educational trajectory in teaching mathematics

We agree in general with Ganchev's conclusions about the building math lessons as fundamental classroom experience [ibid]. However, the development of the global educational environment allows organizing math education much closer to the individual specifics of the learner. A convenient interpretation of the cognition trajectory for describing the individual approach is the *individual educational trajectory*. By ***individual educational trajectory*** (IET) we will understand

organizational frame and plan for realization of a medium term educational process that is coherent with the individual specifics of the learner and provides opportunities for the optimal development of his/hers creative potential [Lazarov, 2012].

The concept of IET refers to the educational microcosmos of the student which is immersed into the global educational environment. The design and implementation of the IET is a complex process that includes the following components.

1. Formation of an individual informational environment.
2. Individualization of the didactical resources, including selection of the individual (re)searching instruments.
3. Individualization of setting the educational goal, including flexible approach to achieve it.
4. Individualization of the learning temps, investigation activities, layout style.
5. Taking into account the individual reflexive abilities and self-organization aptitude in searching a synergetic effect. [ibid]

The process of building IET is a step-by-step (iterative) procedure and any of the listed components is supposed to be actualized according to the student's interim achievements. The developing of the entire process goes in two directions, let call them vertical and horizontal. The description of the horizontal movement will be given in the next section of the article.

The vertical movement in the k-th iteration is formed of the following steps.

…(k – 1) →

- A near educational goal (of learning, investigative or research type) is mapped out with respect to the actual knowledge, skills and competences (KSC) of the learner (actual development).
- A (very limited) informational resource is determined which is focused on the goal.
- All needed activities to extend the actual KSC to a level required for reaching the stated goal are performed.
- Student proceeds to the goal in a *specified manner*.
- The achievements are analyzed.

→ (k**+**1) …

The educational goal in the (k**+**1)-th iteration should require learner's KSC among the elaborated ones in the k-th iteration. So the starting KSC of the (k**+**1)-th iteration are among a subset of the achieved KSC in the k-th one. The transition

(k**–**1) → (k) → (k**+**1)

supposes actualization of the components (1)-(5). This vertical movement along IET could be illustrated as shown in the figure 4 – the architecture of our model in building IET reminds of the architecture of the skyscraper Taipei 101. Further we will call this model DMT, which is an abbreviation of *didactical model Taipei*.

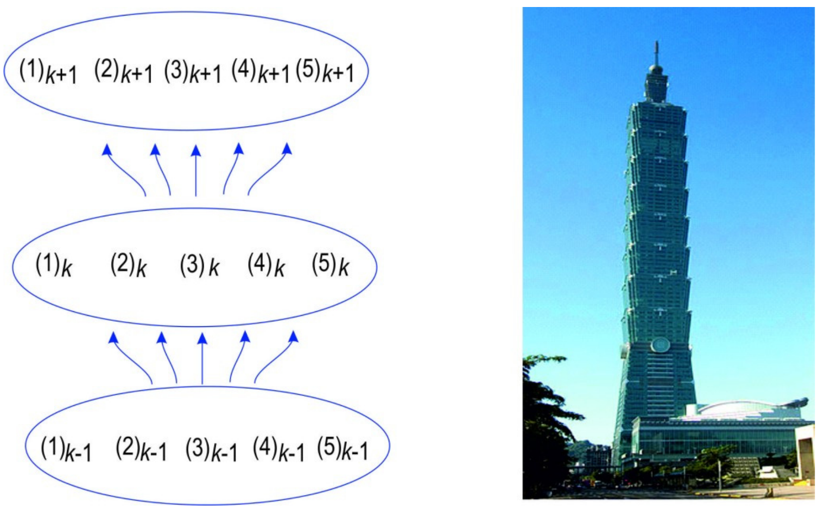

Figure 4. Iterative structure of building IET and the skyscraper Taipei 101 (the photo is taken from http://en.wikipedia.org/wiki/File:Taipei101.portrait.altonthompson.jpg).

As one can see DMT provides a frame to turn education in a manageable process of flexible type. The flexibility of the architecture in vertical perspective is guaranteed by the revision of the components (1) – (5) on any iteration. Additional flexibility is added by the horizontal structure of the DMT.

4. Basic concepts related to the horizontal movement

The process of building IET in any iteration could be described with some auxiliary models. It refers to the horizontal movement in DMT structure. Further we are going to clarify the 'specified manner' pointed in the fourth bullet from the description of the k-th iteration in the above section. This manner can vary significantly due to the development of the student's KSC. The auxiliary didactical models we apply are under the influence of the cybernetic ones from the Consequence Driven Systems (CDS) theory [Ackovska, 2010]. The basic concepts we are going to use in any of our auxiliary models refer to some analogues in CDS.

First let us say that any particular student acts in a ***local behavioral environment*** (LBE) which is a complex socio-economical and cultural structure. For our purposes we consider LBE including:

- people related to the student's behavior (teachers, parents, classmates etc.);
- institutions that organize education and creative work (school, clubs etc.);
- events that provides opportunities to manifest the achievements (tournaments, conferences etc.);
- system of values that form the cultural context of the student (motivation factors, anticipation about the future professional realization etc.).

DMT restricts the local behavioral environment to the listed components as far as they affect most directly the student's educational behavior but we are clear about the simplification of the reality. For instance our model neglects the emotional status of the student that sometimes is crucial in taking decision. The change of LBE depends on the student development. E.g. if (s)he succeeds with a project (s)he can attend some conferences to present it where (s)he can: meet new people, join new clubs, see new opportunities for future professional realization etc.

The ***learning and creativity interface*** (LCI) is the next composite concept that appears in the auxiliary models. We consider LCI as the triad (EC,SS,ER) where EC stands for the ***educational context***, SS is the ***Socratic style interaction*** between the teacher and the student, ER are the ***educational resources***. The EC is the refraction of the LBE through the educational goal, a set of signals that are sent from LBE ingredients and affect the student's educational activeness related with the stated goal. SS is that component that describes the personal site of the teaching-learning process, communications between the student and the teacher related to the educational goal. ER include the didactical and technical tools, sources of information etc. that are implemented during the study and research activities. All three ingredients of the triad LCI ensures the real time development during the movement along an IET. They interact and the momentary magnitude of any part depends on the momentary status of the educational process.

The last composite concept that appears in our general scheme is the triad KSC of ***knowledge, skills and competences***. Here we skip the details and refer to [Winterton et al., 2006].

The architecture of the horizontal movement is shown in the figure 5. In general this architecture represents the most common case. However, some particular applications deserve to be considered separately.

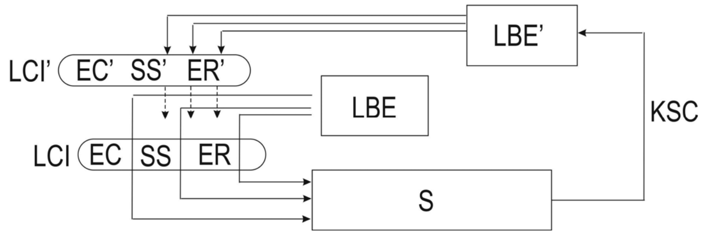

Figure 5. Architecture of the horizontal movement in IET.

5. Some special cases of the general auxiliary model

The general auxiliary model of the horizontal movement allows modifications in any particular stage in building IET. The taxonomy we present below looks like a didactical replica of the CDS one given by Ackovska [ibid]. However, an important difference between the cybernetics and didactics is the nature of the learning agent: our model considers the student as a creative person who is responsible for his/her education. Another difference is the matter of the interaction between the key players: the Socratic style in our model significantly differs from the instructional style in CDS.

Context-independent model. This is an auxiliary model for education with restricted dependence on EC. It is convenient for the starting levels of the IET when the educational goal is in a new area for the student. Student is supposed to generate initial knowledge and to elaborate basic skills in a field that allows stating the educational goal. In such a situation the impact of the LBE onto educational goal is indirect and very limited, thus it can be neglected. The main LCI components are ER and SS. At the output the KSC are also without considerable third component – competences. The Socratic style could be realized mainly as giving advices and encouragement. The educational resources should be user-friendly: the reading materials should be easy; the software should be relevant to the educational status of the student.

EC considering Models. These are auxiliary models that are suitable in various stages of building IET. We are going to introduce them in order of simplicity.

At the final stage of a particular IET the student-teacher interaction could be diminished according to the formed student's competence of synthetic type. The student has got high level skills and is able to create new knowledge. The educational context stimulates him/her to achieve some results relevant to a LBI that includes prestigious events as contests or conferences. Student's reflexive abilities and self-organization aptitude allows him/her to state educational goals by himself/herself. The Socratic style takes the form of equipollent communication. So we can speak for **LCI with restricted teacher's assistance.**

The interim stages of IET are described with auxiliary models in which the components of the LCI interact in full scale. A common feature of all such models is that SS provides a kind of reinforcement. More often than not the reinforcement is **context-dependent**. The teacher has closer look at the student's efforts and organizes an inquiry that allows the student to achieve the stated KSC. The ER should be relevant to the EC. Let us give some examples.

- Tuning the didactical tools: if a problem is too hard at the moment the teacher can decompose it; if a concept is not clear in general case, some particular cases to be proposed for consideration; if a construction is not properly designed, a deductive analyze to be set for discussion etc.
- Fitting the informational sources: if a book is too hard for understanding, it could be substituted with some easier articles; if the student refers to a web-site which is not reliable, a web-search for other sources to be organized etc.

- Conducting the progress in computer skills: organizing a step-by-step procedure for studying the computer program with properly graded examples.

The social microcosmos is of great importance. EC is directly affected from the other students' treatment of the achievements of our student's. Success or failure in some activities could change the student's self-confidence and could influence the educational goals. This leads to some changes in the didactical resources which the teacher applies. Thus we can speak about **consequence dependent model**.

The accommodation of the teacher's interventions along the IET leads to another models. Teacher can give advices or reinforcement immediately but only if it is necessary. In this case we speak about **tutorial teaching model**. The teacher can launch an intervention after student makes several trials to achieve a particular result. This is **delayed intervention model**.

6. How the theory works

In this section we are going to give an idea about how the above models were applied in building a particular IET. We present briefly some parts of a case study carried out in 2011/2012 scholastic year with a 12th grade student. This student (further we call her PL) was involved in a middle term activity during 2010/2011 scholastic year when she performed rather successfully in generalizing an idea from the math tournament Chernorizetz Hrabar. PL was interested also in Wasan geometry and this gave us reason to state the following educational goal: *to develop further student's synthetic competence via studying some sangaku constructions for dynamic stability*. The plan was some dynamic-geometry applets to be designed that illustrate geometrical properties pointed in sagaku problems which are invariant when the parameters of the construction are changed; the results to be proved by methods from the Euclidean and/or analytical geometry; the calculations to be performed using computer algebra system if necessary; eventually a report to be prepared and presented at some conferences.

The first iteration in building this particular IET. Let us explain how the ground floor of the DMT was furnished.

1) The informational environment was restricted to a set of issues of the Bulgarian magazine Education of Mathematics and Informatics where Jordan Tabov edited the column *Problems of the issue* dedicated to sangaku problems.
2) Socratic style was applied every time some questions arise; a considerable amount of short term activities were organized to fill up some gaps in geometry mainly connected with loci.
3) The interim educational goal was: the interface of GeoGebra to be studied, some simple constructions to be performed and examined about dynamic stability; some appropriate pictures of sangaku tablets to be captured from the WWW (no upgrade of some competences was planned). The research process was focused on extracting some common constructions from the sangaku problems among the *problems of the issue*.
4) The temps of learning and elaborating skills were intensive due to the deadlines for submitting reports for an annual student conference. As a matter of fact the in being synthetic competence of the student allows such intensity: she has already passed a similar training process previous year.
5) We also refer to the student's self-control due to the reflexive abilities and self-organization aptitude shown in the previous period.

PL activities during this period were not considerably affected by the education in school or relations with classmates; the student's motivation was not connected with some tests or examines; the eventual participation in conferences was far enough to have direct impact at this stage. Thus the influence of the LBE was very limited and we can speak that this part of the IET was context independent.

The second iteration. The just described preparatory work drifted a lot of collateral information. Despite the recommended literature was limited, the number of sources used by the student raised, the math methods that appears in the sources were also too many to be studied deeper. So the first steps in the IET on this (second) stage of the DMT were to clean up the collateral information and methods and to focus on the potentially suitable ones. Let us give an example.

A large number of the sangaku problems deal with circles and this is why naturally appear constructions that remain the arbelos. Such constructions are well studied by Fukagava, Okumura, Watanabe and other authors (a comprehensive list of books and articles is given in [Watanabe, 2011]). But in some sangaku tablets appear also constructions with circles and ellipse. The figure compound by an ellipse and two internally touching it circles (called by us ***elliptic arbelos***) is not studied separately (as far as we know). Moreover, some interesting properties of such figures were observed. Since the usual method to simplify an arbelos-likely construction is inversion, PL was recommended to study the elliptic arbelos applying inversion. But the image of the inverted ellipse occurs analytical curve of 4^{th} grade – the method did not work. So we decide to skip elaborating skills in inversion and to focus on analytical methods. Using analytical methods instead of the classical geometry proofs is usually regarded as prejudice of deduction. But the deductive side was not neglected: the proofs of the basic properties of the elliptic arbelos were performed by PL in traditional Euclidean style.

Now let us highlight the role of the context with the next example. PL had the opportunity to contact directly Prof. Jordan Tabov who is the editor of the column *Problem of the issue*. He gave her genuine Japanese books dedicated to Wasan geometry and encouraged her for further study of the topic. The additional informational resources were important. But more important was the external positive opinion from an international expert – this was another powerful stimulus for PL, who recognized the importance of her activity.

The advices and reinforcement in this stage were given in Socratic style applying the tutorial auxiliary model. The interim goals were covered successfully and the results obtained were enough to be reported at two consecutive math conferences for school students.

The third iteration. The opportunity to present her work at conference changed significantly the EC. Further activities planned were connected with the layout of the content. But during polishing details a lot of questions appeared and answering these questions leaded to a considerable upgrade of PL's synthetic competence. Let us give an example. The proof of one of the main theorems about the elliptic arbelos required an inequality to be verified. After several failures in solving the inequality by hand PL turned to graphical methods and managed to give strong reasons about the solutions of the inequality. (Later an analytical solution was given to her by Prof. Nikolay Nikolov in a private communication). We decided to pay less attention to the technical skills in solving equations, inequalities etc. Instead of this PL turned her efforts to interpret the results obtained about the equations with a computer algebra system. On this stage in building this IET we applied delayed intervention model.

The next iterations in the DMT refer to the application of the consequence dependent model. Performing successfully at several conferences and contests PL became more confident and even when she did not solve some of the stated problems she proposed conclusive graphical or numerical arguments. Her LBI included some students who share the same interests and views. The next moment is indicative: PL needed a kind of animation to demonstrate the change of the radius of the incircle when the elliptic arbelos changes its type from intersecting to tangential and then to non-intersecting. She did not know how to make this animation. She wrote a question in the GeoGebra Forum and received several suggestions. PL was fascinated by the helpfulness of the international math community. When she asked the persons who helped her about their names to write an official acknowledgement she was surprised with their modesty – no one considered such cooperation as something extraordinary that need acknowledgement.

After several months of studying and working under supervision by the author of this paper PL became enough independent and she got her own view on the theory of the elliptic arbelos. Our IET completed. We think that the educational goal was achieved in general. PL was competent to study a mathematical object using different approaches including advanced analytical, graphical and numerical methods, to prepare a report and presentation, to present the results to competent auditory, to discuss different sides of the findings.

7. Final remarks

The taxonomy of the learning paradigms proposed by Ackovska [ibid] sketches 10 cybernetic models each one approved in some technological processes, i.e. any model works in a real life situation. The educational process of a human is more specific and needs special cares that take into account the personality of the learner. Our experience shows that any attempt to apply strict regulations in teaching-learning process diminishes the effectiveness of the education. On the contrary, a teaching-learning of flexible type gives better outcomes in general but needs quite larger arsenal of didactical instruments than any hard didactical technology. The IET we had build used a variety of auxiliary didactical models that allow us to react adequately in any particular stage. The Ganchev's ideas (pointed in the second section) are more technological and serve the in-class math education directed to covering some educational standards. In contrast to this the IET is directed to a broader field – building a competence of synthetic type (*synthetic competence*) in which the math knowledge (math key competence) is just a component. Via IET we lose in size of the target group but we gain the deepness of the knowledge, skills and competences build.

The complete design of IET is not possible to be made in advance. The collateral information should be cleaned up in any step, i.e. to be neglected when stating the next educational goal. The Socratic style of teacher-student interaction should be coherent with the local behavior environment and the adequate auxiliary didactical analogs of the cybernetic models help to put in practice a desired individual educational trajectory.

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