Computer Aided Engineering and Simulation

RESEARCH OF ACCURACY OF GUARANTEED OPERAYTING TIME PREDICTION BY FRACTILE ZONES METHOD

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Abstract: *This paper describes a method of forecasting the guaranteed operating time using the quantile zones. There are presented an equations and graphics for calculations of the guaranteed time operating error.*

Key words: *Reliability, guaranteed time, parameters' drifts, quasi-deterministic, quantile zones.*

ACM Classification Keywords: *B.8.1 Hardware - Performance and Reliability - Reliability, Testing, and Fault-Tolerance*

Introduction

It is known that among of the precision devices output parameters drift process a great part is non-stationary processes, which are variables not only the expectation and standard deviation of instantaneous values, but also dependent on the placement time interval correlation function. These processes are not ergodic is enough inertia in time irreversibility is determined by the gradual accumulation of changes, in turn, leads to smooth changes in the nature of mathematical expectation. The mean square deviation of a random component is much less than the tolerance field, because such processes are called kind of quasidetermined. Probabilistic prediction of parametric reliability of products can be made by prediction changes in the density distribution f[x(ti)] and determining on this basis since the possible options to achieve the threshold with some confidence probability. Known that to describe the drift of mathematical expectation use exponential or linear model, and to describe changes in standard deviation use linear model. Using these dependencies can build models of change over time fractile parameter values, and they help to make a prediction of the reliability with given probability of finding the parameter in the prescribed range. Guaranteed uptime T_{gar} and its variation is determined by the points of intersection of the functions of mathematical expectation m(t), upper fractile $\alpha_1(t)$ and lower fractile $\alpha_2(t)$ in setted tolerance levels Δ_1 and Δ_2. This T_{gar} is defined as the average time without a parametric failure, t_1 and t_2 respectively its minimum and maximum values. Dispersion since losing parametric reliability ΔT defined period between t_1 and t_2. This method is relatively simple and accurate, and allows to determine not only for 50% resource, but to other probability need only identify the fractile. But not always such processes can be processed using this method. In some cases, ΔT can be overwhelming, and sometimes altogether uncertain. As a result, it is necessary to study the method for its suitability in a particular case. Develop some criteria which would allow to check on the suitability of the method during the minimum number of calculations to statistical data processing. [Bobalo, 1996]

Describing of the method

If the experimental values of the parameter x_s, s = 1 .. k, in the intervals Δt_i, i = 1 .. n, then in each such period of products state is characterized by the density $f[x(t_i)]$. The probability of preservation efficiency is determined by the equations:

$$\mathrm{P}(\mathrm{t_i}) = P\{x(t_i) > \Delta_1\} = \int_{\Delta_1}^{\infty} f[x(t_i)]dx . \quad (1)$$

$$\mathrm{P}(\mathrm{t_i}) = P\{x(t_1) < \Delta_2\} = \int_{-\infty}^{\Delta_2} f[x(t_i)]dx . \quad (2)$$

$x_{hr} = \Delta$ is the limit (permissible) value of x(t); $f[x(t_i)]$ - density distribution of instantaneous values of the parameter in the range Δt_i. Accordingly, the probabilistic prediction of parametric reliability of products can be made by changes forecasting in the density distribution $f[x(t_i)]$ and determining on this basis since the possible options to achieve the threshold. Guaranteed uptime of T_{gar} and its variation is determined by the intersections of functions m(t), $\alpha_1(t)$ and $\alpha_2(t)$ tolerance in levels of Δ_1 and Δ_2:

$$\begin{gathered} T_{gar} = \arg\left|m(t) = \Delta_1\right|; \;\; T_{gar} = \arg\left|m(t) = \Delta_2\right|; \\ t_1 = \arg\left|\alpha_1(t) = \Delta_1\right|; \;\; t_1 = \arg\left|\alpha_2(t) = \Delta_2\right|; \\ t_2 = \arg\left|\alpha_1(t) = \Delta_2\right|; \;\; t_2 = \arg\left|\alpha_2(t) = \Delta_1\right|. \end{gathered} \quad (3)$$

Device guaranteed uptime error ΔT estimated by equation:

$$\begin{gathered} \Delta T_{1gar} = T_{gar} - t_1 , \\ \Delta T_{2gar} = t_2 - T_{gar} , \\ \Delta T = \Delta T_{1gar} + \Delta T_{2gar} . \end{gathered} \quad (4)$$

[Nedostup, 1998]

The study of guaranteed operating time error depending on the slope coefficients

For linear parameter drift processes

For linear change of mathematical expectation and standard deviation of change construct the following equations.

$$m(t) = m_0(1 - k_1 t), \quad (5)$$

$$\begin{gathered} \alpha_1(t) = m(t) - u\sigma_0 - uk_2 t , \\ \alpha_2(t) = m(t) + u\sigma_0 + uk_2 t . \end{gathered} \quad (6)$$

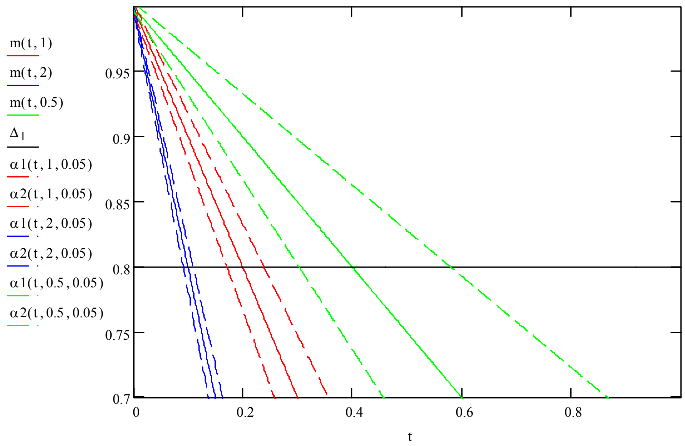

Fig. 1. Graph of the mathematical expectation and fractile for 3 different values of k_1 for fixed value of k_2.

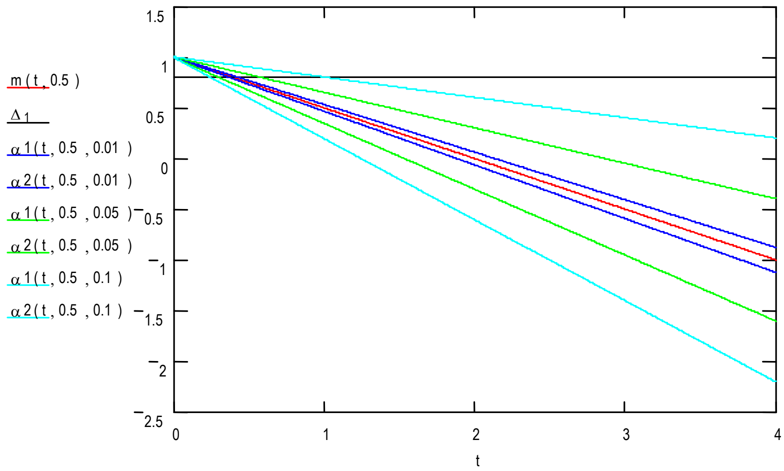

Fig. 2. Graph of the mathematical expectation and fractile for 3 different values of k_2 for fixed value of k_1.

From the pictures can be noted that the increased k1 do that the time difference between both fractile interception is reduced, with increased k_1 ΔT error decreases but decreases and guaranteed time (intentionally increase k_1 lead only to deterioration of circumstances). And there is another dependency k_2: increasing k_2 ΔT error increases and the value of guaranteed time is independent of k_2. Therefore advisable to try to reduce the k_2. Equating formed to determine the intersection points with the tolerance level T_{gar}, t_1, t_2.

$$m(t) = m_0\left(1 - k_1 T_{gar}\right) = \Delta_1 , \tag{7}$$

$$\alpha_1(t) = m(t_1) - u\sigma_0 - uk_2 t_1 = \Delta_1 ,$$
$$\alpha_2(t) = m(t_2) + u\sigma_0 + uk_2 t_2 = \Delta_1 . \tag{8}$$

And:

$$T_{gar} = \frac{m_0 - \Delta_1}{m_0 k_1} ,$$
$$t_1 = \frac{m_0 - u\sigma_0 - \Delta_1}{m_0 k_1 + uk_2} , \tag{9}$$

$$t_2 = \frac{m_0 + u\sigma_0 - \Delta_1}{m_0 k_1 - u k_2}.$$

Errors are defined as follows:

$$\Delta T_{1gar} = T_{gar} - t_1 = \left[\frac{m_0 - \Delta_1}{m_0 k_1} - \frac{m_0 - u\sigma_0 - \Delta_1}{m_0 k_1 + u k_2}\right],$$
$$\Delta T_{2gar} = t_2 - T_{gar} = \left[\frac{m_0 - \Delta_1}{m_0 k_1} - \frac{m_0 + u\sigma_0 - \Delta_1}{m_0 k_1 - u k_2}\right]. \quad (10)$$

Losing parametric reliability time dispersion ΔT determined by the sum:

$$\Delta T = \Delta T_{1gar} + \Delta T_{2gar}. \quad (11)$$

To determine the influence coefficients k_1, k_2 for guaranteed time prediction error construct graph family of the error depending for the first fractile and the second fractile (ΔT_{1gar}, ΔT_{2gar}).

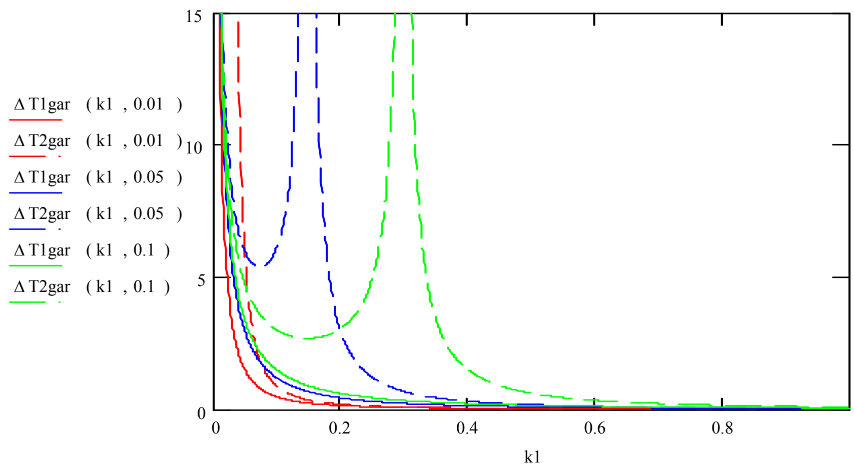

Fig. 3. Graph of guaranteed time error equation on k_1 factor.

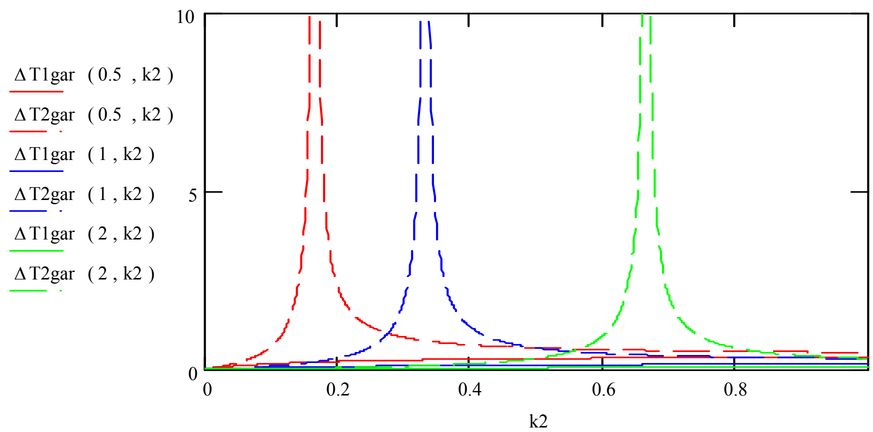

Fig. 4. Graph of guaranteed time error equation on k_2 factor.

Graph in fig. 3 show that for certain values of k_1 error ΔT_{2gar} behaves unclear due to the fact that in this case ΔT_{2gar} is unclear and since a value of k_1 error starts to decrease from infinity. That can be determined that an increase in the coefficient k_1 decrease error but the decrease a guaranteed time to, in addition there is the extent to which k_1 less error is uncertain, this limit is lower at lower values of k_2. The graph in fig. 4 illustrate that an increase of the coefficient k_2 increases measurement errors ΔT_{1gar}, ΔT_{2gar} so that ΔT_{1gar} tends to t when m(t)=Δ, and tends to infinity ΔT_{2gar} when approaching k_2 to a certain extent which is greater at larger values of k_1.

Behaviour of error indicated that there are some limits to the values of k_1 and k_2. Based on the nature of relationships and graphic material received is below these values k_1 and k_2 in which fractile α_2 becomes equal to a constant:

$$\alpha_2(t) = m_0 - m_0 k_1 t + u\sigma_0 + uk_2 t = c\,. \tag{12}$$

This constant is easy to find it is the initial fractile value :

$$c = \alpha_2(0) = m_0 + u\sigma_0\ . \tag{13}$$

Where is the following condition: $-m_0k_1t$ must compensate uk_2t. The result is the equation:

$$-m_0 k_1 t = uk_2 t\,.$$

To get the limit equation will reduce t:

$$\frac{k_1}{k_2} = \frac{-u}{m_0}\,. \tag{14}$$

For the growing dependence of all remains the same and the results are similar:

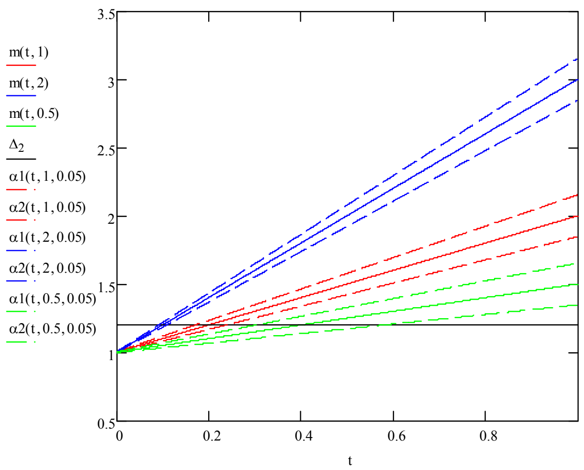

Fig. 5. Graph of the mathematical expectation and fractile for 3 different values of k_1 for fixed value of k_2.

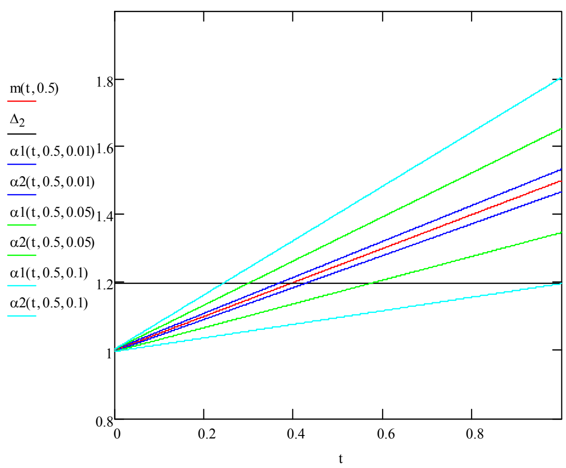

Fig. 6. Graph of the mathematical expectation and fractile for 3 different values of k_2 for fixed value of k_1.

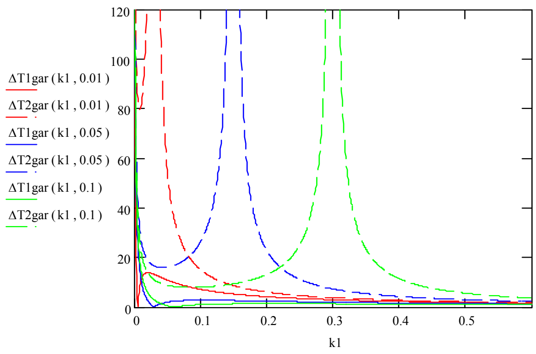

Fig. 7. Graph of guaranteed time error equation on k_1 factor.

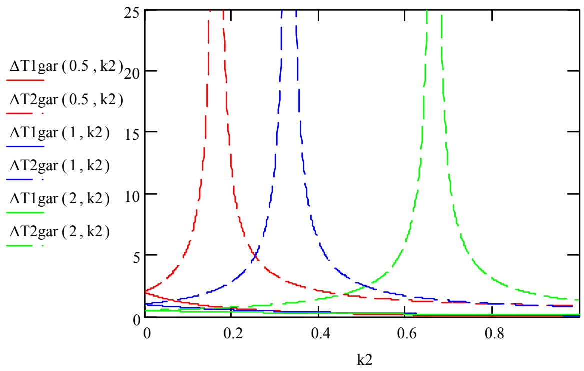

Fig. 4. Graph of guaranteed time error equation on k_2 factor.

The limit for the case of the growing nature of drift takes the form:

$$\frac{k_1}{k_2} = \frac{u}{m_0}. \tag{15}$$

For the exponential parameter drift processes

Consider the descending process. In the linear approximation moments T_{gar}, t_1, t_2 are determined from the equations:

$$m(T_{gar}) = m_0 \exp(-k_1 T_{gar}) = \Delta_1; \tag{16}$$

$$\alpha_1(t) = m_0 \exp(-k_1 t_1) - u\sigma_0 - uk_2 t_1 = \Delta_1;$$
$$\alpha_2(t) = m_0 \exp(-k_1 t_2) + u\sigma_0 + uk_2 t_2 = \Delta_1. \tag{17}$$

Solving the first equation we get:

$$T_{gar} = \ln\left(\frac{m_0}{\Delta_1}\right)^{1/k_1}. \tag{18}$$

The second and third equation is transcendental relative to t1, t2precise methods of their solutions do not exist. Therefore, schedule exponent $e^{-k_1 t_2}$ in series:

$$\exp(-k_1 t) = 1 - k_1 t + \frac{(-k_1 t)^2}{2!} + \frac{(-k_1 t)^3}{3!} + \ldots \tag{19}$$

This alternating series as known is converge for any $k_1 t_1$. Where as $k_1 t_1$ may be small, can be neglected components with the powers number begin from second. That is consider that [Korn, 1984]:

$$\exp(-k_1 t) = 1 - k_1 t.$$

And:

$$\varepsilon = \sum_{n=2}^{\infty} \frac{(-k_1 t)^n}{n!}. \tag{20}$$

Then equation (17) can be rewritten as:

$$m_0\left(1-k_1 T_{gar}\right)-u\sigma_0-uk_2 T_{gar} \approx \Delta_1 . \qquad (21)$$

And:

$$t_1 \approx \frac{m_0-\Delta_1-u\sigma_0}{m_0 k_1+uk_2},$$

similarly:

$$t_2 \approx \frac{-m_0+\Delta_1-u\sigma_0}{-m_0 k_1+uk_2}.$$

So:

$$\begin{aligned}\Delta T_{1gar} &= T_{gar}-t_1 \approx \ln\left(\frac{m_0}{\Delta_1}\right)^{1/k_1}-\frac{m_0-\Delta_1-u\sigma_0}{m_0 k_1+uk_2};\\ \Delta T_{2gar} &= t_2-T_{gar} \approx \ln\left(\frac{m_0}{\Delta_1}\right)^{1/k_1}-\frac{-m_0+\Delta_1-u\sigma_0}{-m_0 k_1+uk_2}.\end{aligned} \qquad (22)$$

And ΔT is determined by the sum:

$$\Delta T=\Delta T_{1gar}+\Delta T_{2gar}.$$

In the case of the growing exponential and linear approximation we obtain the relation:

$$m(t)=m_0\left[1-\exp(-k_1 T_{gar})\right]=\Delta_2; \qquad (23)$$

$$\begin{aligned}\alpha_1(t) &= m_0\left[1-\exp(-k_1 t_2)\right]-u\sigma_0-uk_2 t_2=\Delta_2;\\ \alpha_2(t) &= m_0\left[1-\exp(-k_1 t_1)\right]+u\sigma_0+uk_2 t_1=\Delta_2.\end{aligned} \qquad (24)$$

Guaranteed time errors is calculated by the equations:

$$\Delta T_{1gar}=T_{gar}-t_1 \approx \ln\left(\frac{m_0}{m_0-\Delta_2}\right)^{1/k_1}-\frac{\Delta_2-u\sigma_0}{-m_0 k_1+uk_2}; \qquad (25)$$

$$\Delta T_{2gar}=t_2-T_{gar} \approx \ln\left(\frac{m_0}{\Delta_1}\right)^{1/k_1}-\frac{\Delta_2+u\sigma_0}{m_0 k_1+uk_2}. \qquad (26)$$

Losing parametric reliability time dispersion, as in the previous case, determined by the sum:

$$\Delta T=\Delta T_{1gar}+\Delta T_{2gar}.$$

Now consider the case of quadratic approximation of decreasing and increasing exponentials, according to preliminary considerations will describe the exponential quadratic equation. Then:

$$\exp\left(-k_1 t\right)=1-k_1 t+\frac{(-k_1 t)^2}{2!}. \qquad (27)$$

The equations of mathematical expectation m(t) and fractiles $\alpha_1(t)$ and $\alpha_2(t)$ when descending exponentially take the form:

$$m(t) = m_0 \exp(-k_1 T_{gar}) = \Delta_1 ; \tag{28}$$

$$\alpha_1(t) = m_0\left[1 - k_1 t_1 + \frac{(-k_1 t_1)^2}{2!}\right] - u\sigma_0 - uk_2 t_1 = \Delta_1 ;$$

$$\alpha_2(t) = m_0\left[1 - k_1 t_2 + \frac{(-k_1 t_2)^2}{2!}\right] + u\sigma_0 + uk_2 t_2 = \Delta_1 . \tag{29}$$

The solution of these equations T_{gar}, t_1, t_2 are :

$$T_{gar} = \ln\left(\frac{m_0}{\Delta_1}\right)^{1/k_1} ; \tag{30}$$

$$t_1 \approx \frac{(m_0 k_1 + uk_2) \pm \sqrt{(m_0 k_1 + uk_2)^2 - 2(m_0 - u\sigma_0 - \Delta_1) m_0 k_1^2}}{m_0 k_1^2} ;$$

$$t_2 \approx \frac{(m_0 k_1 + uk_2) \pm \sqrt{(m_0 k_1 - uk_2)^2 - 2(m_0 + u\sigma_0 - \Delta_1) m_0 k_1^2}}{m_0 k_1^2} . \tag{31}$$

Changing the output setting for the growing exponential law describes by the dependencies:

$$m(t) = m_0 \left[1 - \exp(-k_1 T_{gar})\right] = \Delta_2 ; \tag{32}$$

$$\alpha_1(t) = m_0\left[1 - \left(1 - k_1 t_2 + \frac{(-k_1 t_2)^2}{2!}\right)\right] - u\sigma_0 - uk_2 t_2 = \Delta_2 ;$$

$$\alpha_2(t) = m_0\left[1 - \left(1 - k_1 t_1 + \frac{(-k_1 t_1)^2}{2!}\right)\right] + u\sigma_0 + uk_2 t_1 = \Delta_2 , \tag{33}$$

$$t_1 \approx \frac{(m_0 k_1 + uk_2) \pm \sqrt{(m_0 k_1 + uk_2)^2 - 2(\Delta_2 - u\sigma_0) m_0 k_1^2}}{m_0 k_1^2} ; \tag{34}$$

$$t_2 \approx \frac{(m_0 k_1 + uk_2) \pm \sqrt{(m_0 k_1 - uk_2)^2 - 2(\Delta_2 + u\sigma_0) m_0 k_1^2}}{m_0 k_1^2} . \tag{34}$$

The choice of linear or quadratic approximation of the average change in alue during the operation carried out by comparing the approximation error with the requirements for the accuracy of prediction reliability. These dependences reflect the relationship between the reliability of the devices, the initial values of parameters and patterns of change in service. It is clear that among the characteristics most subject to management during the initial values of parameters that can be set rationally considering reasonable manufacturing tolerances. Based on the above equations are built dependency graphs of mathematical expectation and fractiles for different values of slope k_1 and k_2 (Fig. 9, 10).

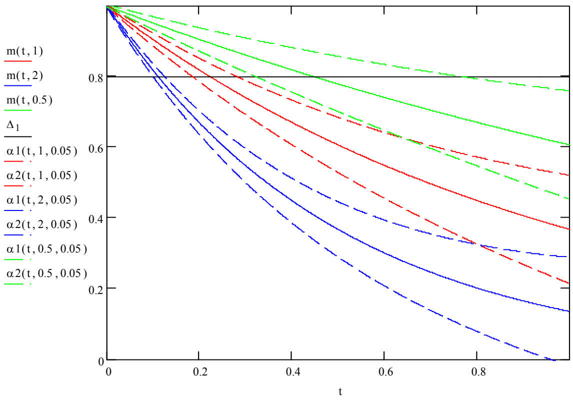

Fig. 9. Graph of the mathematical expectation and fractiles for different values of k_1 for fixed values of k_2.

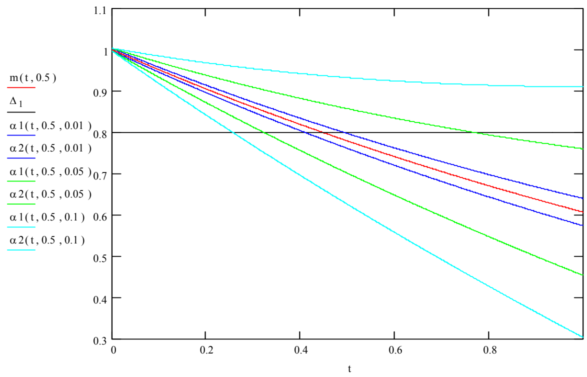

Fig. 10. Graph of the mathematical expectation and fractiles for different values of k_2 for fixed values of k_1.

On fig. 9 and 10 shown that there are times when fractile not cross tolerance level changing its direction to reversed. It is similar situation as with linear drift. To determine the moment of time in which fractile change their direction build derivatives of each fractile. From mathematics we know that the derivative shows tangent angle function, so when derivative crossed with zero level the fractile is a change direction (Fig. 11).

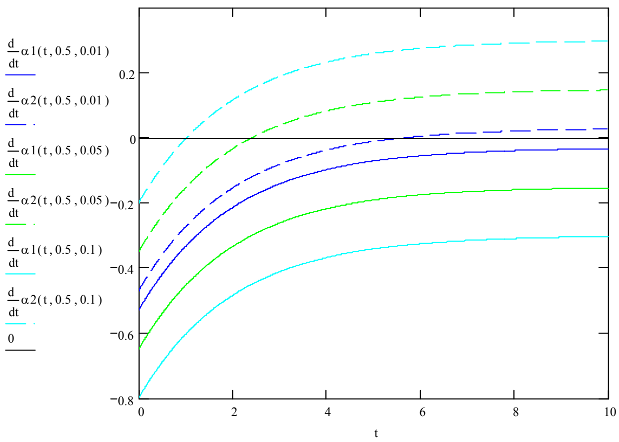

Fig. 11. Family of derivatives on fractiles α_1, α_2.

On fig. 11 shown time points when fractile α_2 begin to increase ($t \approx 1, 1.5, 5.5$), but if the fractile not crossed the tolerance level to it time it will not cross never.

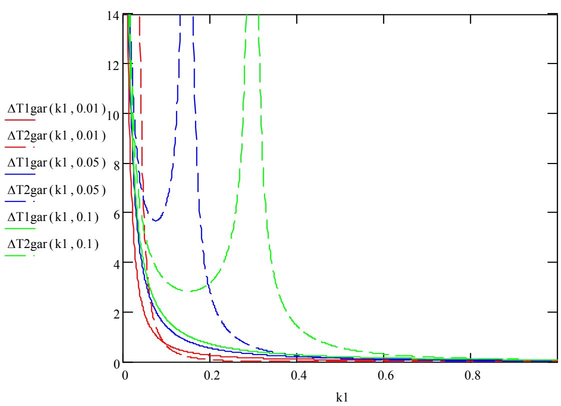

Fig. 12. Graph of guaranteed time error equation on k_1 factor.

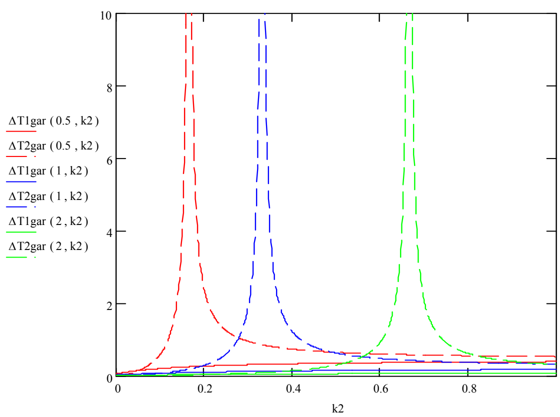

Fig. 13. Graph of guaranteed time error equation on k_2 factor.

For exponential character of drift error behaves similarly to linear drift. We can identify the limit equation from the following equation systems:

$$\begin{cases} \alpha_2(t)' = 0 \\ \alpha_2(t) = \Delta_1 \end{cases}. \tag{35}$$

But formed equation is transcendent.

Similar properties have a growing process. Schedules for the growing process are shown in fig. 14, 15, 16, 17.

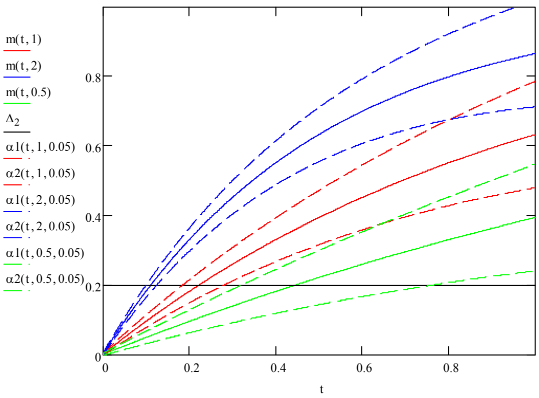

Fig.14. Graph of the mathematical expectation and fractile for different values of k_1 at fixed values of k_2.

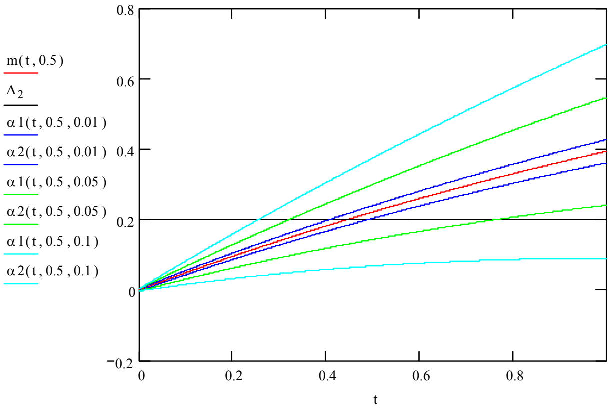

Fig. 15. Graph of the mathematical expectation and fractile for different values of k_2 for fixed values f k_1.

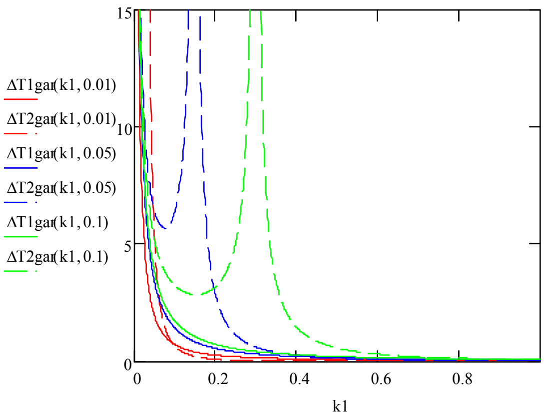

Fig. 16. Graph of guaranteed time error equation on k_1 factor.

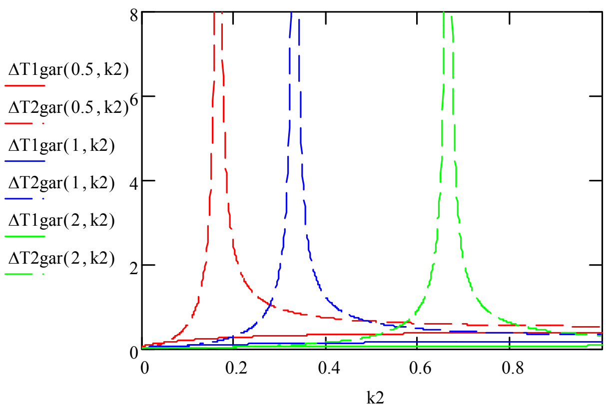

Fig. 17. Graph of guaranteed time error equation on k_2 factor.

Conclusion

As a result of the research is presented the some boundary conditions ("limit") to which method is suitable and effectiveness. For linear drift parameter limit is determined and clearly established (14, 15), and in the case of exponential nature of the drift parameter limit becomes transcendental form and therefore requires the solution of the transcendent equation for each case is derived from (35). Also found that reducing the error of guaranteed time desired is the increase in steepness parameter drift and drift reducing the slope standard deviation, but in terms of reliability necessary to reduce both the coefficients of steepness becouse is advisable to reduce the slope coefficient of standard deviation drift .

Bibliography

[Nedostup, 1998] Л.А.Недоступ, М.Д.Кіселичник, Ю.Я.Бобало. Основи надійності радіоелектронних пристроїв. За редакцією д-ра техн. наук, проф. Л.А.Недоступа. Львів. Видавництво Державного університету "Львівська політехніка",1998 рік.

[Bobalo, 1996] Бобало Ю.Я., Кіселичник М.Д., Недоступ Л.А. Системний аналіз якості виробництва прецизійної радіоелектронної апаратури. За редакцією проф. Недоступа Л.А. Монографія. Львів. ДУ «ЛП». 1996р.

[Korn, 1984] Г.Корн, Т.Корн. Справочник по математике для научных работников и инженеров. М.: "Наука", 1984 г.

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