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## A MULTI-LEVEL FUZZY LOGIC SCHEME FOR MODELING COMPLEX PROBLEMS

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**Abstract:** *In this paper, we propose to extend traditional Fuzzy Logic(FL) techniques by creating a second set of inference rules (and the corresponding inference mechanism) in order to effectively produce results exploitable by decision makers. We present a formal definition of a multi-level FL scheme and its application in the development of: i) a break-even inference machine and ii) a legal argumentation system. The proposed scheme is shown to be useful in modeling systems where intermediary fuzzy concepts come to play, and it could be applied in several problems analog to the ones presented in this study.*

**Keywords:** *Multilevel Fuzzy Logic, Knowledge Based Systems, Legal Argumentation System.*

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### Introduction

FL began in 1965 with the work of Lofti Zadeh; Zadeh aimed to propose a mathematic formalism for leading with imprecise concepts, like high risk, low speed, high cost, etc. Nowadays, FL is being applied in several areas as medicine, engineering, management, law, gesture recognition, etc. Knowledge based systems built on FL could be applied to non-linear or non-defined problems, because it remains one of those few techniques able to model expert knowledge even if it is ambiguous. [Reiter, 1980], [Garmendi, 2010].

However, FL systems follow a well known paradigm of implementation, where developers have a unique set of fuzzy variables and a corresponding set of inference rules, which are not always applicable to real problems, [Casali, 2002], [Bourcier, 2003], [Trillas, 1992]. For instance, solutions to model the break-even in subsidy health systems, or to model the statement on legal argumentation, have proven to be extremely difficult to realize with traditional FL techniques, due to the presence of intermediary and multi-level fuzzy concepts. In these type of systems, the first level refers to uncertainty and concepts part of the nature of the problem. For example, in the break-even system, concepts that model the relation between the cost given by the government, as well as the market price, both need to be fuzzified. [P. Cohen, 1983], [bar, 2001], [Perez, 2005]

On the other hand, in the argumentation system, concepts like aggravating and mitigating facts also require fuzzification. Nevertheless, after acquiring all the costs (e.g., in the break-even system) and the facts (e.g., in the argumentation system), these have to be counted and processed to infer whether we are in a break-even situation or not (break-even system), and whether the person is guilty or innocent (argumentation system).

Also, the decision is not based on crisp sets, but rather on others fuzzy concepts, [Trillas, 1992] for example: with the break-even system, several costs might be equivalent in both government and market price, or few government costs might be more expensive than market price; similarly with the argumentation system, many aggravating facts could emerge and minimum mitigating facts could possibly occur. Such systems remain extremely difficult to model using traditional (single-level) FL schemes.

The present paper have been organized as follows, section 2 present an overview of knowledge-based systems and fuzzy logic, section 3 shows the law and legal reasoning, section 4 present the approach and finally sections 5 shows the conclusions and future works.

### Knowledge-based Systems and Fuzzy Logic

Fuzzy Logic was developed by first time in 1965 to process and handle information specially to represent mathematically uncertainty no probabilistic, which is present for example in non-linear problems [Funkhouser et al., 2005], [Giarratano, 2001]. One of its main advantages over the classic logic is that it allows to reproduce the reasoning, considering the certainty of a preposition such as a level given; so, if logic is the science of the formal and normative principles of the reasoning, fuzzy logic concerns to the formal and normative principles of the approximate reasoning and considering the classic logic such as his limit [Brio, 2006], [Doyle, 1979]. Figure 1 shows the fuzzy and classic logic.

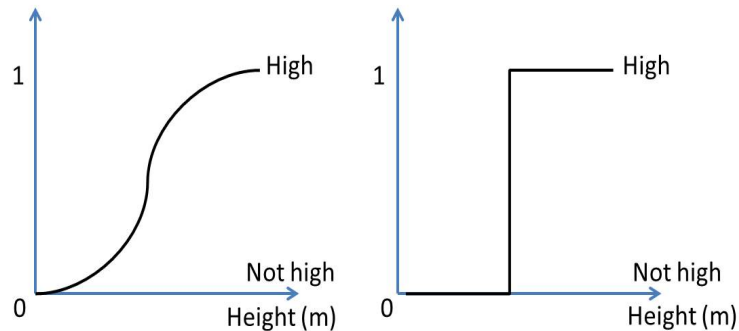


Fig. 1. Fuzzy and Classical Logic

Fuzzy logic use rules that shows the relation between antecedent and consequent; usually it uses IF-THEN rules, however, these rules can be also inferred; this work use the Modus Ponens as inference rule as it is shows as follow:

Knowledge	: If x is A, then y is B
Fact	: x is A
Conclusion	: y is B

After the fuzzyfication process is done, it is necessary a defuzzyfication to convert the outputs of the fuzzyfication in an linguistic approximation or an arithmetic value that represent the fuzzy set.

### Law and Legal Reasoning

The law is a cognitive technology, i.e. a set of practices described as a product of intellectual activities and the acquisition of knowledge capable of being processed by computer. These technologies aim to do more intelligible rationalization of legal knowledge.

#### 3.1. Expositive part

In this part, it is found the description of the main fact, furthermore is identified to the guilty, the juridical and factual imputation, penal consequence, the facts pleaded by the defense.

#### 3.2. Preamble

This is the valuative part of the judgment, i.e. the charge and discharge facts, the law and the criminal law to apply.

#### 3.3. Resolute part

The last part of the judgment explains the declaration of the criminal liability. The penalty is calculated in this part.

**Proposal**

The present paper proposes the application of the multilevel fuzzy logic in a legal argumentation system based on an extension of the FL techniques by creating a two level fuzzyfication process. The figures 2 and 3 shows the general schema of the process and the inference machine, respectively.

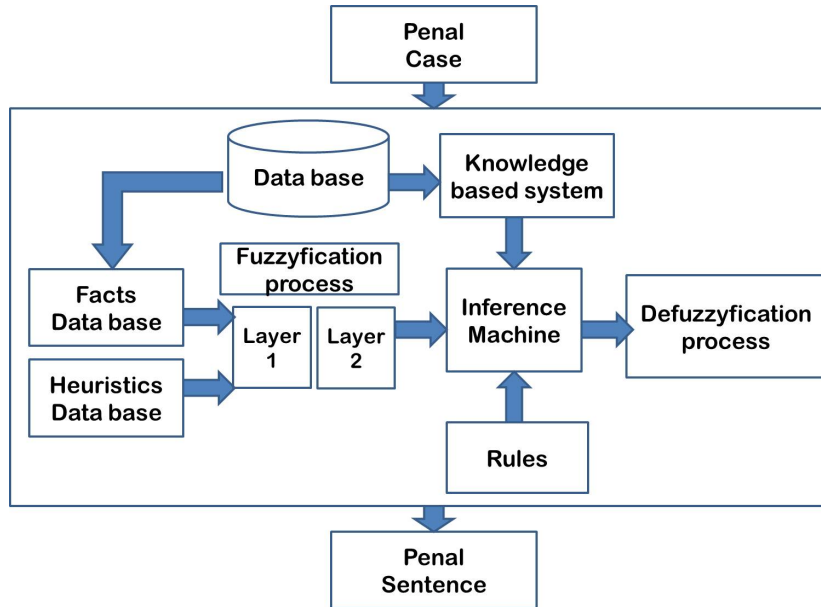


Fig. 2. Approach Schema

As can be seen in the Figure 2, the input of the system is given by the case to judge, it means, the description of the principal fact and which will be storage such information and inferences. After the facts and information are storage, the system determine if the case is or not of penal type; this will be done based on the search of key words (i.e. die) in the feature vector of the case to judge.

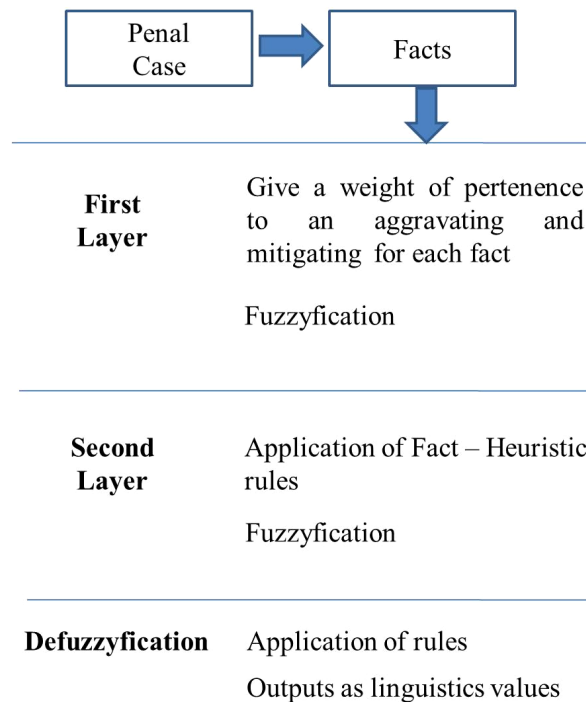


Fig. 3. Inference Machine

#### 4.1. First Layer

In this layer the judge gives a weight, from 1 to 10, for each fact; this in order to establish the degree of membership in the aggravating and mitigating<sup>1</sup> fuzzy sets as it is shown in the figure 4. In order to compute the membership function of the sets; it is used the triangular function being 5 the break point, which means that a fact with 5 as value of degree of membership is considered as normal and does not affect the assessment of the case to judge because this is the cross point between both sets.

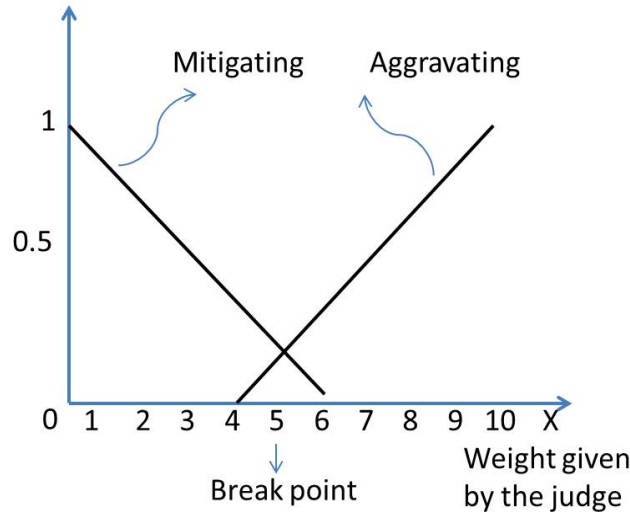


Fig. 4. Aggravating and mitigating fuzzy sets

After the assessment is given to each fact, the level of guilt is calculated using the following rules:

$$\begin{aligned} &\text{if } \mu_{\text{MITIGATING}}(\text{Fact}_i) > \mu_{\text{AGGRAVATING}}(\text{Fact}_i), \\ &\text{then } FR1 = \sum_{i=1}^n \text{Fact}_i * (\mu_{\text{MITIGATING}}(\text{Fact}_i)) + FR1 \\ &\text{else } FR2 = \sum_{i=1}^n \text{Fact}_i * (\mu_{\text{AGGRAVATING}}(\text{Fact}_i)) + FR2 \end{aligned}$$

Where F R1 is the sum of mitigating facts, F R2 is the sum of aggravating facts and n is the number of facts. These output values (F R1 and F R2) in this layer, will be the inputs in the next layer.

After all the facts are assessment, it is necessary to determine which ones are relevant evidence, in consequence, an  $\alpha$  cut that allow to make the filter process was applied. In this case, the value used was 0,6 ( $\alpha 0,6$ ) in view of it had a high rate of incidence in the selected data. In this way, it is possible to determine if the mitigating or aggravating facts are higher.

#### 4.2. Second Layer

A difference of the layer before, this layer make an assessment of the heuristics related to the facts; as in the case before, a weight from 1 to 10 will be given for each one, as shows the figure 5, and a fuzzyfication process will be make.

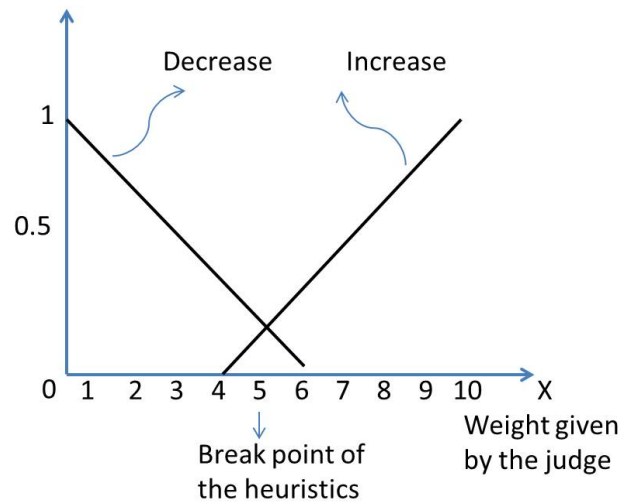


Fig. 5. Fuzzy sets respect to heuristics

This will allow to establish the sentence of the defendant applying the rules Fact - Heuristic (FH), where the heuristic chosen will be applied to the fact and will fulfill the height of the fuzzy set, in other words, the higher value will be chosen to calculate the sentence according to the higher fuzzy set, it is shown as follow:

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if       $\sum FH_i * (\mu_{DECREASE} (FH_i)) > \sum FH_i * (\mu_{INCREASE} (FH_i))$ 
then    $\max(\mu_{DECREASE} (FH))$ 
else    $\max(\mu_{INCREASE} (FH))$ 

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As can be observed the sentence given is based in the relevant facts related with the computation of the resolutive part, where a sentence is given according to all the facts related with the heuristics and applying the following rules.

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if       $\max, \mu_{INCREASE} (FH)$ 
then    $JudgmentGiven = MinSentence + \mu_{INCREASE} * \omega$ 
else    $JudgmentGiven = MinSentence + \mu_{DECREASE} * \omega$ 

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Where MinSentence and MaxSentence are the minimum and maximum sentence respectively and  $\omega$  is given by:

$$\omega = MaxSentence - MinSentence$$

#### 4.3. Defuzzification

The inference machine is strongly related with the defuzzification process, because in this layer the system shows the outputs as linguistics values which are obtained applying the inference machine. The rules used in this process are:

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if       $(F R1 > F R2)$ ,
then    $Subsumption - Culpability = Innocent$ 
else    $Subsumption - Culpability = Guilty$ 

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The subsumption of the crime is also done by the inference machine using the following rules:

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if      • The main fact is of penal type AND
        • type of act = Degree of enforcement AND
        • Fact description = Tentative OR

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- Factdescription = Consumption AND
- Facttype = Participation OR
- Factdescription = Autor OR
- Factdescription = Participate

then     Subsumption – Quality of Typical = Typical  
else     Subsumption – Quality of Typical = Atypical

if     • Subsumption – Quality of Typical = Typical AND

- FH – Type = Justification AND
- Decrease level = 1

then     Subsumption – Antijustified = Justified  
else     Subsumption – Unlawful = Antijustified

After these rules, the fact rules are also included, so:

if     • Subsumption – Unlawful = Unlawful AND

- Subsumption – Quality of Typical = Typical AND
- Subsumption – Culpability = Guilty

then     Subsumption – Crime = Positive  
else     Subsumption – Crime = Negative

At this point all the considerative part that must be write and the sentence is done.

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## Conclusions and Future Works

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The process of make a decision in a judgment and find the best statement for the sentence is an important and complex task, in this sense, it is necessary the use of a system that can help in this process, making it essayer, having a database of historic data for consult previous cases and using fuzzy logic that helps to establish the sentence, based in the evaluation of the facts. In this sense, an inference machine with two fuzzyfication levels is necessary in order to reach better results, also, the uncertainty of the data does not allow the application of classic logic.

The proposed system is also flexible allowing the judge to give a valuation for each fact; likewise, the use of an  $\alpha$  cut of 0.6 in the system is also important to distinguish between the relevant and irrelevant facts, avoiding that unnecessary facts appears in the sentence. Finally and as future works it is proposed the implementation of this system as well as a dynamic cut, that can be computed by the system automatically.

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