BUSINESS INTELLIGENCE SYSTEMS

J. FORRESTER'S MODEL OF WORLD DYNAMICS AND ITS DEVELOPMENT (REVIEW)

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Abstract: *At far 1970 the elite Roman Club asked prof. J. Forrester from MIT to develop a model of world dynamics. Speaking world dynamics we mean the dynamic interactivity of the main macro economical variables. The 1-st version of the model named "World-1" was presented in 4 weeks and next year the corrected version "World-2" was accepted as the classical J. Forrester's model. In spite of its long history the J. Forrester model retains its actuality being the basis for modern models. In the paper we consider the principal of system dynamic, criticism of the classical model, and the new models developed by the J. Forrester's followers. We consider also adjacent areas and open problems related with world dynamics.*

Keywords: *world dynamics, non lineal dynamics, J. Forrester model*

ACM Classification Keywords: *I.2.m Miscellaneous*

Introduction

1.1. The beginning of J. Forrester's model

The elite Roman Club is non-governmental organization, which joins political und scientific personalities and is working on modeling World Crisis. At 1970 the Roman Club asked prof. J. Forrester from the Massachusetts Institute of Technology (MIT) to develop a model of world dynamics. Speaking world dynamics we mean the dynamic interactivity of the main macro-economical variables. The first version of the model named "World-1" was presented in 4 weeks and next year the corrected version "World-2" was accepted as the classical J. Forrester model. In spite of its long history the J. Forrester model retains its actuality being the basis for modern models, so we can say, that this model is actual nowadays in spite of its elderly age. The models based on J. Forrester's approach can predict crises and sometimes help to avoid it. So, such models are very important.

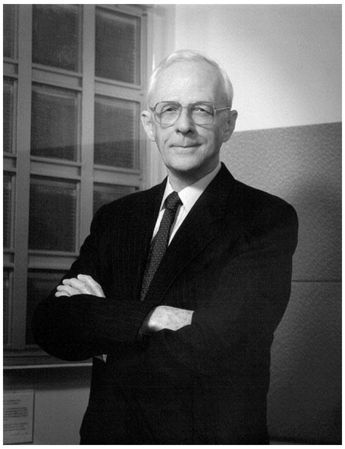

Fig.1. Prof. J. Forrester

There are many reviews in literature associated with these models [Makhov, 2003; Makhov,2005; Malineckiy, 2010, etc.]. We tried to collect shortly the principal positions reflected in the mentioned publications.

1.2. Principles of system dynamics

System dynamics is based on two main principles. First of all, equations of the same type are prepared for all variables:

$$\frac{dy}{dt} = y^{+} - y^{-}. \quad (1)$$

Here y^+ is the positive rate of variable change (it includes all factors related with increase of the variable y), y^- is the negative rate of variable change (it includes all factors related with decrease of the variable y).

Thereafter it is supposed that all rates (the positive and negative ones) could be presented in the form of function compositions, which depend on one factor (combination of main variables):

$$y^{\pm} = g(y_1, y_2, \dots, y_n) = f(F_1, F_2, \dots, F_k) = f_1(F_1) f_2(F_2) \dots f_k(F_k). \quad (2)$$

1.3. J. Forrester's model

J. Forrester in his work saw five main problems, which could provoke the World Crises. It is overpopulation of our planet, lack of basis resources, critical level of pollution, food shortages and industrialization and the related industrial growth. He tied a single variable with each of these issues. So, we have a five-level system, which define the structure of the system:

• Population (P).

• Pollution (Z).

• Natural resources (R).

• Fixed capital (K).

• Capital investment in agriculture fraction (X).

For the system level J. Forrester proposed the following differential equation:

$$\frac{dP}{dt} = P(c_B B_C B_P B_F B_Z - c_D D_C D_P D_F D_Z) \tag{3}$$

$$\frac{dK}{dt} = c_K P K_C - \frac{K}{T_K} \tag{4}$$

$$\frac{dX}{dt} = \frac{X_F X_Q - X}{T_X} \tag{5}$$

$$\frac{dZ}{dt} = P Z_K - \frac{Z}{T_Z} \tag{6}$$

$$\frac{dR}{dt} = -P R_C \tag{7}$$

Here he used tabulated functions (with linear interpolation) and constants: c_B=0,04 (normal fertility rate), c_D=0,028 (normal death rate), c_K=0,05 (normal rate of capital), T_K=40 (time of depreciation main funds), T_X=15 (time of depreciation agricultural funds), t_N=1970 (initial year), P_N=3,6$\cdot$ 10^9(population in initial year), X_N=0,3 (capital investment ratio in agriculture in initial year).

Initial data are: $t_0 = 1900$, $P_0 = 1{,}65 \cdot 10^9$, $K_0 = 0{,}4 \cdot 10^9$, $X_0 = 0{,}2$, $Z_0 = 0{,}2 \cdot 10^9$, $R_0 = 900 \cdot 10^9$.

Standard pollution Z_N is numerically equal to the population. R_0 is a rate of mineral resources consumption. It is taken under the two conditions: a) it is equal the rate in 1970 b) this rate is enough for natural resources would be sufficient for 250 years

The behavior of the model parameters is shown in figure1. It can be seen that after a period of growth, the population ***P*** begins to decline since 2020. Non-renewable natural resources in 2100 are less than 30% of the original stock. Pollution reaches its maximum in 2050, about 6 (more precisely, 5.8) times exceeding the standard level, then it drops due to the general decline of industry and population decline. Level of life reaches its maximum about 2000, and then decreases.

Such a behavior is a consequence of resource depletion. Less natural resources less the level of life. The latter causes increasing death rate and reduces investments. And, finally, we have a sharp population decline and a fall of industrial production (funds). J. Forrester tried to change the original settings in order to avoid the crisis, but every time the crisis arose.

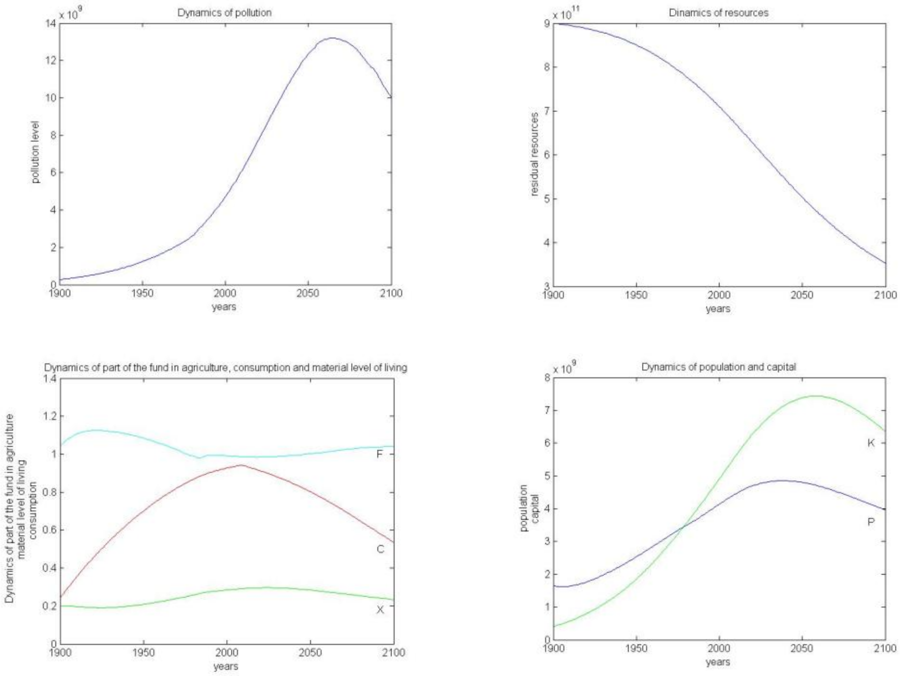

Fig.2. The behavior of the main macro economical variables

Critical analysis of J. Forrester model

Like any model, the global dynamics of J. Forrester has some limitations [Forrester, 2003]. Many researchers have noted it in their publications.

2.1. Critics by Moiseev

Moiseev's notes:

1. Methodological flaw: there are no conservation laws of material flow in the model that would reflect the economic balance
2. STP is not considered although it changes the nature of production and increasing productivity
3. There are no social mechanisms for the distribution of wealth in the model

4. Unreasonable nature of the scalar concept of a "quality of life".
5. The model is too rigid: once started "correct" model we then obtain a model of a completely different nature
6. Some table functions are incorrect (in the sense of the nature of relationships): factors may obtain other values
7. Mechanistic understanding of the concept of equilibrium, which J. Forrester recommends to prevent the crisis.

2.2. Critics by Egorov

Egorov [Egorov, 1980] and Gelovani [Gelovani, 1977] note:

1. Disaggregated model across regions is needed.
2. There is no possibility of conscious human impact on the process of development
3. Lack of any control due to the rigidity of the investment distribution
4. It is incorrect to give recommendations how to prevent a serious crisis without analysis of all possible actions
5. "Stability" offered by J. Forrester is unreachable in the framework of his rigid model

Many researchers have criticized the Forrester's model of world dynamics, mainly due to its rigidity and lack of technological factors.

Models of J. Forrester's followers

3.1. Meadows' model

One of the works, which can be considered as a development of J. Forrester's model is the work of Meadows [Meadows, 2007]. He is a pupil of J. Forrester. Meadows expanded the model having introduced several system levels in addition to those of J. Forrester. The integration of processes was completed at the same time interval. Modeling showed that on the qualitative level the new model was similar to the of J. Forrester model. But, unfortunately, Meadows had no enough quantity of data (according to him, he had only 0.1% of the required information). The results agree with Forrester's model because, restoring the missing data, Meadows was guided by the model of his teacher. By the way the Meadows' model also showed that a crisis was inevitable due to resource depletion.

3.2. Egorov's model

Modeling world dynamics allowed to formulate the simple conclusion: if the current trends of social development is inevitable then it will be a serious crisis in relations between humans and the environment. Growth can not continue indefinitely, sooner or later it will stop. But how will this growth stop? The answer of the model did not satisfy J. Forrester and he introduced the concept of global equilibrium. However it proved to be impossible to achieve a steady state for all variables in the framework of his model. The group of researchers headed by Egorov from the Keldysh Institute of Applied Mathematics proposed a way to achieve such a balance (a stationary mode of model functioning). The main positions [Egorov, 1980] are the following:

- it is possible to recover (recycle) resources
- there are ways to reduce pollution
- there is a control of capital distribution between agriculture, resource recovery and struggle with pollution
- model is modified during its functioning
- an optimal control problem is formulated and resolved for the “corrected” model with the appropriated criterion of model quality

Speaking on the “language of equations” these positions mean the following:

a. Right part of the equation for resources is added by an additional summand:

$$\frac{dR}{dt} = -PR_C + \frac{K}{Cr}U_R \tag{8}$$

Here U_R is the part of funds, which is used for recovering resources, and C_R is the cost of repairing the unit of resource

b. The right part of the equation for pollution is added by an additional summand:

$$\frac{dZ}{dt} = PZ_K - \frac{Z}{T_Z} - \frac{K}{C_Z}U_Z \tag{9}$$

Here U_Z is the part of funds to be directed for the struggle with pollution and C_Z is the cost of cleaning the unit of pollution.

c. The right part of the equation for agricultural is added by the factor (1+U_X):

$$\frac{dX}{dt} = \frac{(1+U_X)X_F X_Q - X}{T_X} \tag{10}$$

d. The level of life is calculated by the new formula:

$$C = K_p \frac{1 - X - U_R - U_Z}{1 - X_0} \tag{11}$$

Egorov has shown that the new model has non-zero steady-state solutions. Therefore with the controlled model the world system can avoid a crisis in the sense of J. Forrester.

3.3. Matrosov’s model

Another approach is proposed by a group of Matrosov [Matrosov, 1999; Matrosova 1999]. Here are the principle positions related with their model:

- the dynamics of biomass of vegetation is introduced (the model contains a "control parameter": coefficient of pollution influence on biomass)
- the single-product macroeconomic models are introduced (there are an explicit GDP), which form an expanded sector of the economy
- the time-dependent STP is introduced as an index of average productivity of labor
- the factor of political tensions and management is introduced
- we introduce the appropriate factors and parameters for modified equations

The stationary solutions for the model were calculated and they proved their stability.

Thus, the authors [Matrosov, 1999; Matrosova 1999] completed a large-scale modification of J. Forrester model. In essence, it was a new model. However, serious doubts concerning the adequacy of this model appeared. Forrester's model is "self-sufficient": the factors and parameters in it were chosen to simulate the dynamics of the past. The Matrosov's model [Matrosov, 1999; Matrosova 1999] has no this property. On the other hand, the model has a number of advantages. One of them is the detailed economical sector.

3.4. Makhov's model

The last modification of the model was proposed by one of the authors [Makhov, 2010]. His model contains the following parameters: population ***N***, energy reserves ***R***, fixed assets, similar to J. Forrester's model. But, unlike the mentioned models the new model does not include agriculture and pollution. They are replaced by the level of technology ***T*** and the education ***E***. The agriculture sector depends on the energy and capital that is it is not an independent factor. For the same reason pollution is not considered separately: it depends on territory and resources. Besides, the affect of pollution on the environment was not studied well.

The demographic equation is based on the well-known model [Makhov, 2010]:

$$\frac{dN}{dt} = c_N N\left(1 - \frac{M}{M_{max}}\right) \tag{12}$$

The equation for capital can be written as (it is a result of logical reasoning):

$$\frac{dK}{dt} = I - \mu K \tag{13}$$

Here ***I*** is the cross (fixed) capital formation, ***μ*** is the retirement rate.

The analysis of data over the past 40 years shows that the fractional amount of final consumption and cross (fixed) capital formation is only slightly deviated from 1 (see Fig.3).

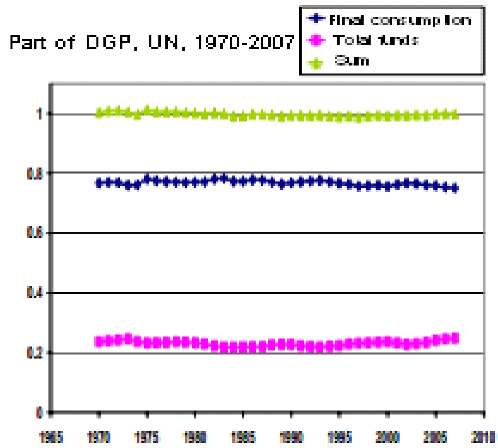

Fig.3. Parts of final consumption and total funds in GDP, and its sum

The equation for resources looks like:

$$\frac{dR}{dt} = -R_D + R_P \tag{14}$$

Here R_D is production and R_P is replenishment of resources. Energy-related part is assumed to be directly proportional to the GDP (see Fig.4).

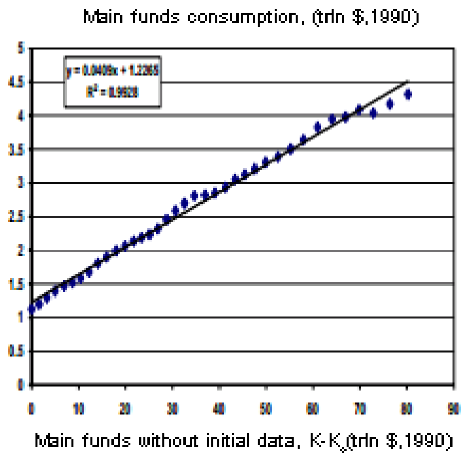

Fig.4. The main funds consumption as a function of the total gross savings in capital (the integral of gross) for 1970-2006

The equations for the sectors of education and technology are built empirically. A dependence of the population on the capital and technology is given by the Cobb-Douglas function.

Thus, the Makhov's model of world dynamics [Makhov, 2010] is presented in the form:

$$\frac{dN}{dt} = c_N N\left(1 - \frac{g_c Y}{N M_{max}}\right) \quad (15)$$

$$\frac{dK}{dt} = (1 - g_c)Y - \mu K \quad (16)$$

$$\frac{dR}{dt} = -k_R Y + R_p \quad (17)$$

$$\frac{dT}{dt} = \alpha(c_T (g_T T)^m E^n - T) \quad (18)$$

$$\frac{dE}{dt} = \beta(a + bT - E) \quad (19)$$

$$k_R = k_0 - k_1(t - t_0) \quad (20)$$

$$Y = TK^a N^{1-a} \quad (21)$$

Conclusion

In the paper we presented the review of Forrester's model and the models of his followers.

- 40 years passed but researchers continue to study and develop various models of world dynamics. Such a fact has a simple explanation: crises come now and they will come in future
- In order to build any own model of world dynamics one should study first of all the classical J. Forrester model

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