METRIC TENSOR AS DEGREE OF COHERENCE IN THE DYNAMICAL ORGANIZATION OF THE CENTRAL NERVOUS SYSTEM

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Abstract: *The mechanism by which neuronal networks dynamically organize and differentiate during development is a salient issue concerning neurogenesis. This central event reflects a topological order and its metrizability. One important parameter in this process concerns the role of tremor and intrinsic electrical properties of neurons [Llinàs 1988] from a different in the developmental organization of Central Nervous System (CNS), which we now would like to develop more formally. While tremor is usually considered an undesirable parameter in the generation of coordinated movement it is actually essential in efficient motor execution and reflects fundamental intrinsic neuronal electrophysiological oscillation. In addition, we propose, such intrinsic properties contribute to organize the neuronal connectivity that results in the development of internal coordinate reference systems. Thus the degree of coherence in the oscillatory activities of neuron can be interpreted as embodying a metric tensor of non-Euclidean space that produce topological order associated to CNS development.*

Keywords: *Degree of coherence, Metric tensor, Nervous System, Intrinsic Oscillation, Functional Geometry*

Introduction

Central Nervous System (CNS) activity has evolved to represent the external world into internal parameters that ultimately organize and modulate its interactivity with the surrounding environment. Such evolution forged neuronal ensembles, as interconnected groups (areas) capable of internally defining specific intrinsic parameters reflecting significant properties of the world addressable by motor commands. Intrinsic characters and internal trends fundamentally modulate the activity in such areas. Thus the parameters which internalize the world are fundamentally self reliant and internally consistent, and represent a secondary emission imbedded reality, but only on their own terms and parameters. Such intrinsically driven set of compartments internalize the specific sensory representations that support movement interaction with the external reality. Thus, sensors by acting in unison (co-variants) represented external properties to be transformed into guided action on the world (contra-variants), implementing appropriate motor specification. It was originally proposed [Pellionisz and Llinas 1982; Pellionisz and Llinas 1985] that such correspondences between covariant vectors (for sensory effects) and contravariant quantities (for motor preparation and effectuation) constituted structures of metric spaces, which provide the brain with internal models of the world. The dimension of time was added[Roy and Llinás 1988] to these metrics: in that work stochastic metrics represented chaos at local levels that stabilize smooth oscillations at greater scale [Leznik et al 2002] giving quanta of information at 40Hz. However, in this paper the main issue is what aspects of cell function serve as the linkage between single cell properties and the development of neuronal nets as it relates to the interaction with the external world.

The prevailing view at this time is that such organized structure is basically a product of genetic information unfolded in time and of epigenetic variables relating to (i) position, (ii) axonal growth, (iii) target location. Edelman [Edelman 1984] added one more step towards understanding the process of organization i.e. the specific regulation of cellular movement through gated adhesiveness followed by *selective stabilization* of synaptic convertibility [Changeux and Danchin 1976]. One of us [Llinàs 1984a] critically analyzed the situation and raised questions concerning (1) what further parameters other than biochemical are selective and (2) what mechanisms implement this selection. It was proposed, at that time a new hypothesis, relating these two aspects of network genesis to intrinsic neuronal auto-rhythmicity that could be considered as playing a central role in early organization of nerve nets [Llinàs 1984b]. Autorhythmicity and cell to cell communication, through electronic coupling, could in addition generate synchronicity of firing by entrainment of co-existing intrinsic oscillatory properties. These properties along with the chemical synaptic transmission present in early embryos, were thus proposed as an essential electrical substrate of the organization of brain circuits [Llinàs 1984a]. From a different perspective it is known that controlled tremor can be considered the basis for motricity [Llinàs 1991]. The essential feature of tremor is a kind of activity that is sustained and regular. Tremor is usually considered as undesirable property in the organization of coordinated movement but, in fact, it is essential in the organization of the internal coordinate reference systems.

It is well-known that during development embryos go through distinct stages of tremor and rhythmic twitching. In Parkinson's disease mechanical oscillatory simulation of a finger by external device may induce tremor that irradiates progressively upwards along the limb. The tremor march phenomenon indicates that the different portions of CNS which control limb movement are coupled to each other by the ability to be phase-locked and to resonate with tremor occurring in one of its parts. The mechanism responsible for generating tremor may be considered to be associated to the oscillatory rhythm as coherent properties of CNS neurons. We propose that the degree of coherence depending on intrinsic neuronal oscillation produce topological order to the neuronal networks. This degree of coherence can be identified as Non-Euclidean metric tensor of this topological space associated to CNS. At first the role of tremor in the development of organization of nervous system will be briefly discussed in section II and then the topological order and the metric tensor for the topological space associated to the functional states of CNS will be constructed in section III. Possible implications will be discussed in section IV.

Tremor and Dynamic Organization of Central Nervous System

Coghill in his well known book Anatomy and the Problem of Behavior addressed the neuro-embryological basis of behavior for diverse vertebrates. He tried to find basic principles that would be valid for vertebrates in general. The development of behavior in vertebrates as proposed by Coghill [Coghill 1920] has been opposed by Windle1 [Windle 1940]which emphasizes that the local reflexes are the basic units of behavior. Hamburger [Hamburger 1963] in his comprehensive review critically analyzed the two approaches and the Spontaneous and reflexogenic motility are considered to be as two independent basic ingredients of embryonic behavior. It is also mentioned that the direct observations of motility in case of higher vertebrates the behavior is uncoordinated in the early stage. Bakeoff et al [Bekoff et al 1975] studied $7-7.5$-day chick embryo using electromyography since this stage of chick embryo is close to the inception of overt motility in immature leg. This effort was motivated to find whether or not the motor system is organized at this stage. Their study indicated sensory input is not required for the development of coordinated motor output. Their investigation also showed that embryos go through distinct stages of tremor and rhythmic twitching during development. The essential feature of tremor is that of an activity that is sustained and oscillatory motion of a body part. The early studies show that physiological tremor is related to the rhythmic neural activity of CNS where as pathological tremor be a distortion or amplifications of this rhythmic oscillations. Indeed, neuronal oscillation and resonance determine much of the active movement dynamics of limbs and thus provide, by feedback through the afferent systems, information about the dynamics of the body reference frames [Llinàs 1984a]. For example, in cases of Parkinson's disease, mechanical oscillatory stimulation of a finger by an external device may induce tremor that irradiates progressively upwards, along the limb, in a manner similar to the Jackson's march of motor seizures. It follows that the different portions of the CNS which control limb movement are coupled to each other by the ability to the phase-locked and to resonate with tremors occurring in one of its portions. This indicated the serious possibility that during development CNS may utilize the tremor in establishing an internal geometry of the coordination between the peripheral structures such as muscles and their sensory return to the CNS to establish the dynamics of movement control.

Indeed, since the tremor ultimately returns to the CNS by different afferent receptor connectivity, it may be used, during development, to establish some of the n-dimensional vectorial transformation (in frequency hyperspace) which underlie the implementation of sensory input into coordinated motor output. Thus, the tremor and twitching of a given muscle group and the ability it has to generate sensory feedback by the production of actual movement, by the direct activation of the sensory feedback or by corollary discharge, would ultimately serve to establish the physiologically meaningful connectivity between afferent input and motor output. The importance of this interaction becomes clearer when considering that it occurs very early in development prior to the generation of truly organized movement and could thus serve as an epigenetic organizing influence in determining selective stabilization of neuronal networks [Changeux et al 1973]. This mechanism of central oscillation-giving rise to tremor- in turn generating a sensory stimulus which is feedback to the central oscillator, would ultimately result in the internal modulation of the sensorimotor transformation.

Functional Significance of Tremor

The functional significance of physiological tremor is one of the fascinating issues in neuroscience. Since the physiological tremor is a low amplitude and low frequency [Llinàs 1984a; Llinàs 1991; Coghill 1920; Windle 1940; Hamburger 1963] Hz range oscillation during the maintenance of steady body postures, it is considered a source of unwanted noise in the system, something to be controlled rather than exploited. Such a view was prevalent for a long period. Simultaneously with the development of the ideas described above concerning intrinsic neuronal oscillations and tremor, 1983 Goodman and Kelso [Goodman and Kelso 1983] also explored tremor critically and suggested it could have important functions rather than being just unwanted source of variability. Here, CNS in the process of minimizing this variability (tremor) by neuronal mechanisms capable of plasticity, especially during development, uses adaptive changes that establish the internal geometry required for the transformation of sensory input into a motor output which would be, de facto, in resonance with the bodyï£¡s coordinated dynamics. In addition to tremor organizing sensori-motor transform, oscillatory interaction between other portions of the neurons system must also be at work during development. During development, neurons are known to have auto-rhythmicity. This intrinsic auto-rhymicity is considered to be one of the central mechanism in the early organization of neuronal networks. Auto-rhymicity and cell-to-cell communication through electronic coupling, which characterizes embryonic tissue, can generate synchronicity of firings by entrainment of coexisting intrinsic oscillatory properties. In fact, the degree of coherence of the oscillations or rhythms associated to field potentials or spike trains is shown to be related to degree of communication between structures. These structures are nothing but large-scale network property. The degree of coherence determines the geometrical organization of nervous system.

Degree of Coherence and Metric Tensor

Theory of coherence of electromagnetic filed has been widely discussed both in case of classical and quantum paradigm. The diffraction and interference effects of electromagnetic waves are observed depending on the nature of coherent disturbances. The fully coherent disturbance is a mathematical idealization and in reality the disturbance is intermediate between fully coherent and incoherent to the extent that we can address them as partial coherence. The theory of partial coherence in classical theory has been developed based on Maxwell wave theory and is usually considered a component of the more general information theory concerning statistical dynamics a means to understand information transfer. The theory of coherence is closely connected with classical noise theory and with the theory of quantum fluctuations in the quantum mechanics field. At first we discuss some of the basic concepts useful in understanding the degree of coherence.

Mutual Coherence and Degree of Coherence

The mutual coherence function $\Gamma_{12}(\tau)$ can be defined as

$$\Gamma_{12}(\tau) =< V(\vec{x}_1, t)V * (\vec{x}_2, t+\tau) >$$

where$\vec{x}_1 \& \vec{x}_2$ denotes the position vector, the asterisk a complex conjugate and the bracket be the time average i.e.

$$< F >= lim_{T\rightarrow\infty} \frac{1}{2T} \int_{T}^{T} F dt$$

Above mutual coherence function the following crossing symmetry condition holds :

$$\Gamma_{12}^*(\tau) = \Gamma(-\tau)$$

due to the stationarity property of the disturbances. Considering the normalized mutual coherence, we define the degree of coherence as

$$\gamma_{12}(\tau) = \frac{\Gamma_{12}(\tau)}{\sqrt{\Gamma_{11}(0)}\sqrt{\Gamma_{22}(0)}}$$

where

$$0 \leq |\gamma_{12}(\tau)| \leq 1$$

. $\gamma_{12} = 1$ characterizes the complete coherence and for zero value the complete incoherence. Like the mutual coherence function, the degree of coherence is a function of seven variables : six position coordinates and the time delay coordinate τ. The theory of partial coherence is used to describe the phenomena in real world. Perina [Perina 1969; Perina 1969] investigated the matrix formulation of partial coherence using the Hilbert-Schmidt theory of linear integral equation and its functional theory of coherence. This is similar to Heisenberg matrix and Schrodinger wave formulations of quantum mechanics.The two formulations are shown to be equivalent due to the isomorphism of L_2 spaces(the space of square integrable functions in Lebesgue sense) and I_2 spaces (space of Fourier coefficients of functions from L_2 to a complete orthogonal system of functions). In the similar spirit a covariant formulation of partial coherence has been done using the *quantization* of the object and its image [Gabor 1956] to interpret the degree of coherence as metric tensor in non-Euclidian space called the optical space. This allows to study the partial coherence as a property of the optical space.The intensity can be written in quadratic form as

$$I = \sum_{i,j} \gamma_{ij} u^i u^j$$

where $u^i \equiv u(\xi_i)$, ξ_i are the points of quantization. Here, $\gamma_{ij} \equiv \gamma(u^i, u^j)$. Now one defines a non-Euclidian metric in the usual way[19] using the quadratic form

$$(ds)^2 = \sum_{i,j}^{n} \gamma_{ij} du^i du^j$$

. As $I \geq 0$ this quadratic form is semi positive definite. It is to be noted that the mutual coherence function and the degree of coherence are measurable quantities in contrast to the non-measurable vectors of Maxwell theory. In case of incoherent field, the metric tensor simply reduces to the Euclidian one i.e. $\gamma_{ij} = \delta_{ij}$.

Propagating waves in the cortex and internal geometry

Travelling waves of oscillatory behavior of the nervous system play crucial role in computation and communication between the various subsystems of the brain [Petrov 1961; Ermentrout 2001] including the olfactory system of vertebrates and invertebrates. These waves may take various spatio-temporal forms for example: the membrane voltage profile of neurons at different locations is essentially one-dimensional plane wave. Rubino et al [Rubino et al 2006] studied the information transfer in the motor cortex mediated by propagating waves for high frequency oscillations. They suggested that this information transfer is understood with respect to the spatiotemporal characteristics of the neuronal ensemble oscillatory behavior. The stimulus induced oscillations of the ensemble of neurons and resonances are widely studied in many sensory systems. The synchronization of the oscillatory networks have drawn large attention in various contexts especially the real world neuronal networks. Recently, Motter et al [Motter et al 2005] investigated the synchronization of those complex networks that are considered to have strong heterogeneity in the degree (connectivity) distribution. They have shown that the synchronization of such complex networks depend on the degree and the maximum synchronization is achieved when network is weighted and the overall cost involved in the coupling is minimum. Again here in the oscillatory networks the total strengths of all in-links $k(1-\beta)$ with degree k_i at a node i can be interpreted as a property of the input function of the oscillators rather than the property of the links. We, thus suggest that the tremor in the input makes the overall cost in the coupling minimum so as to achieve maximum synchronicity. In fact, the emerging physical properties from in vitro network are the propagation speeds, synaptic transmission, information creation and capacity. The information speed is estimated based on the wave like associated to the oscillatory neuronal networks. Pathological brain states such as epilepsy are considered to be associated to activity waves. Generally, the neuronal waves are divided into three broad classes [Ermentrout and Pinto 2007]:

- Stationary waves
- Active waves
- Oscillation generated or Phase waves

The phase wave is generated via the intrinsically oscillatory properties where the local het- erogeneity, anisotropy or dynamic instabilities induce phase difference varying over space. The degree of coherence of the waves associated to oscillatory network of neurons is a physically measurable quantity. This degree of coherence can be interpreted as non- Euclidian metric tensor in a space called internal space similar to that described for optical space for electromagnetic waves. This internal space is similar to that proposed some years ago3 associated to CNS. It is to be noted that the optical space in case of electromagnetic waves help us to study the coherence properties of waves themselves in the spirit of Einsteinï£¡s geometric description of the gravitational field. Here, we like to emphasize that the coherence behavior of the oscillatory activity of ensemble of neurons can be studied as a property of the internal geometry as previously mentioned [Roy and Llinás 2011].

Discussions

Oscillations and tremor are generally considered as unwanted variability or noise, present in brains as an unwanted property. Moreover, the limitation put by physical laws in Thermodynamics and Quantum Theory forces us to think of noise as trouble when we try design electronic devices. On the other hand, in living systems, starting from unicellular object like Diatom -Bacteria to more complex ensembles such as neuronal networks noise, as defined in physics is a required component for its functionality. In fact it has been recently pointed out that the Central Nervous System (CNS) depends on noise, which carries no signal information, to enhance signal detection thus emphasizing the fundamental role of noise in information processing of brain. The emphasis of this paper, thus, is geared to underline the significance that intrinsic oscillation and its counterpart, tremor, plays in the development and basic functional organization of the central nervous system. Indeed those two variables play a fundamental role in the development of the actual topological structure associated functioning neuronal networks. Indeed, the degree of coherence associated with the oscillatory networks is centrally determined by the tremor and hence the metric tensor of this topological space. This justifies, in our eyes, our heuristic arguments regarding the role of oscillation in the organization of the nervous system.

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