

Krassimir Markov, Vitalii Velychko, Oleksy Voloshin  
(editors)

**Information Models  
of  
Knowledge**

**ITHEA<sup>®</sup>**  
**KIEV – SOFIA**  
**2010**

**Krassimir Markov, Vitalii Velychko, Oleksy Voloshin (ed.)**

**Information Models of Knowledge**

ITHEA®

Kiev, Ukraine – Sofia, Bulgaria, 2010

ISBN 978-954-16-0048-1

First edition

Recommended for publication by The Scientific Council of the Institute of Information Theories and Applications FOI ITHEA  
ITHEA IBS ISC: 19.

This book maintains articles on actual problems of research and application of information technologies, especially the new approaches, models, algorithms and methods for information modeling of knowledge in: Intelligence metasynthesis and knowledge processing in intelligent systems; Formalisms and methods of knowledge representation; Connectionism and neural nets; System analysis and synthesis; Modelling of the complex artificial systems; Image Processing and Computer Vision; Computer virtual reality; Virtual laboratories for computer-aided design; Decision support systems; Information models of knowledge of and for education; Open social info-educational platforms; Web-based educational information systems; Semantic Web Technologies; Mathematical foundations for information modeling of knowledge; Discrete mathematics; Mathematical methods for research of complex systems.

It is represented that book articles will be interesting for experts in the field of information technologies as well as for practical users.

General Sponsor: Consortium FOI Bulgaria ([www.foibg.com](http://www.foibg.com)).

Printed in Ukraine

**Copyright © 2010 All rights reserved**

© 2010 ITHEA® – Publisher; Sofia, 1000, P.O.B. 775, Bulgaria. [www.ithea.org](http://www.ithea.org) ; e-mail: [info@foibg.com](mailto:info@foibg.com)

© 2010 Krassimir Markov, Vitalii Velychko, Oleksy Voloshin – Editors

© 2010 Ina Markova – Technical editor

© 2010 For all authors in the book.

© ITHEA is a registered trade mark of FOI-COMMERCE Co., Bulgaria

**ISBN 978-954-16-0048-1**

C/o Jusautor, Sofia, 2010

---

---

## Decision Making

---

---

### INDIVIDUALLY OPTIMAL PRINCIPLES OF DISTRIBUTION OF GREENHOUSE GAS EMMISSION QUOTAS

Olexij Voloshin, Sergij Maschenko

**Abstract:** *The problems of climate change mitigation are being analyzed. Different emission quotas distribution mechanisms are observed. The work offers model and method of distribution based on the individual rationality principle.*

**Keywords:** *climate protection, emissions of greenhouse gases, global limit on emissions, distribution methods, Nash equilibrium, individual optimal equilibrium.*

**ACM Classification Keywords:** *I. Computing Methodologies – I.6. Simulation and modelling (Time series analysis) – I.6.8 Types of Simulation – Gaming.*

---

#### Introduction

This work continues the research presented in [Voloshin, 2009,2010]. Climate changes caused by anthropogenic influence on the Earth's atmosphere are one of the most important environmental problems of civilization. The first step in solving this problem was the signing by 154 countries in 1992, the UN Framework Convention on Climate Change (UNFCCC). In 1997 "The Kyoto Protocol to the United Nations Framework Convention on Climate Change" was adopted. Under the Protocol, 37 industrialized countries (called "Annex I countries") commit themselves to a reduction of four greenhouse gases (GHG) (carbon dioxide, methane, nitrous oxide, sulphur hexafluoride) and two groups of gases (hydrofluorocarbons and perfluorocarbons) produced by them, and all member countries give general commitments. Also it provides mechanisms for "trading" emissions. The problem lies in the fact that less than half the countries signatories of the UNFCCC, ratified the Kyoto Protocol (KP). In particular, the KP wasn't signed by the industrial giants - the U.S., China and India. They argue, that KP offers "unfair" principles for assigning emission rights. Discussion of these issues over the last decade has not led to constructive solutions. In particular, the results of the last UN conference on climate changes in December 2009 in Copenhagen proved to be disappointing. The Berlin conference in May 2010, involving representatives from over 100 countries, also did not lead to consensus. In [Voloshin, 2009,2010] statistics on the problem are provided, criteria and indicators, which are accounted (can be taken into account) in the allocation of quotas and the causes of "dead end" situation in solving these problems are analyzed. In [Voloshin, 2009] task of quotas allocation is reduced to the classical problem of cost allocation [Voloshin, 2006], which, by turn, is considered as a cooperative game. The analysis in [Voloshin, 2010] on the basis of the results of recent talks in Copenhagen and Berlin, suggests that the desire of participants to cooperation (regional level, level of development, etc.) is missing. In particular, China and India are ready to participate in activities to implement the KP, if their share of the quotas will be determined based on the amount of emissions per capita. In [Voloshin, 2010] is proposed: first, to base the quotas allocation on the concept of multi-objective optimization using the maximum number of available indicators of economical, environmental, demographical and social development of countries and, secondly, to use a two-tiered procedure for finding a compromise: at the first level the allocation mechanisms have to be agreed, at the second level received allocations have to be analyzed for addressing their integral "quality" which is expressed by the utility function.

### Game Models of Distribution

The last meeting in Copenhagen showed the problem of KP can be the most adequately described by a model of decision making in conditions of complete information with minimal co-operation of players (ideally - in the total absence of cooperation) [Voloshin, 2006]. In such models, the most reasonable is the concept of "Nash equilibrium" - a situation when no player can benefit by changing his or her strategy while the other players keep theirs unchanged. And the most interesting thing is that (as we believe) the world community is in a similar situation! The foregoing is illustrated by a simple example of Samuelson [Samuelson, 2008]. For convenience we will change the numerical values of "profits" of players (without loss of content). Let us consider the game, described in Table 1, where L<sub>1</sub>, L<sub>2</sub> – respectively, the strategy of players 1 and 2, targeted at the low level of contamination; H<sub>1</sub>, H<sub>2</sub> – high level. The cells (situations) A-D show the profit of 1 and 2 players. The only non-cooperative Nash equilibrium (cell D) is inefficient – none of the agents are not able to increase profits by reducing the level of pollution. A shift in cell A (with the same profit players, but with lower levels of contamination) is possible (by Samuelson) by the intervention of "government" (coordinating body) in the establishment of the fines or quotas on emissions. But no government can help, if we change the profit values in cell D, for example, at (100, 100)!

Table 1

1 \ 2	L <sub>2</sub>	H <sub>2</sub>
L <sub>1</sub>	A <b>10; 10</b>	B <b>-5; 15</b>
H <sub>1</sub>	C <b>15; -5</b>	D <b>10; 10</b>

Table 2

1 \ 2	L <sub>2</sub>	H <sub>2</sub>
L <sub>1</sub>	A <b>20; 20</b>	B <b>-5; 15</b>
H <sub>1</sub>	C <b>15; -5</b>	D <b>10; 10</b>

Table 3

1 \ 2	L <sub>2</sub>	H <sub>2</sub>
L <sub>1</sub>	A <b>10; 7</b>	B <b>0; 8</b>
H <sub>1</sub>	C <b>8; 0</b>	D <b>1; 1</b>

Which way out of this situation? First, to obtain "objective" values of "wins" (calculated by taking into account the parameters given in [Voloshin, 2010]), and secondly, it is necessary to depart from the concept of "non-cooperative Nash equilibrium". Rationale for the last assertion comes from the following simple generalization of Samuelson model (Table 2). In this game there are two Nash equilibrium (cells A and D), and in equilibrium A win for each player 2 times greater (with low pollution) and it would seem that the agents could agree on the choice of equilibrium A. However, if in situation A the risk of rejection of each of the players is very high (only losing with 25% gains, while bringing losses to a competitor), then deviation from the situation of D, in principle, is not beneficial to every player. The concept of Nash equilibrium, taking in count "the efficiency of gain" and "the effectiveness of risk" is described in [Harsanyi, 2001]. But this approach does not significantly change the situation of decision making in the KP implementation. It is shown by the following example (Table 3). There is only one Nash equilibrium (cell D), but it is "very ineffective". Cell A gives each player gains substantially greater, but for the player 2 is profitable to deviate from the situation A. How to "force" players choose the situation A? The answer – each player must take into account the interests of another. Formally, the concept of "compromise for the sake of resolving the conflict" (each player chooses a strategy individually, taking into account the interests of other agents) developed in [Maschenko, 2007, 2009], where the principle of "individual optimality" is proposed. It is proved primarily for single-purpose games in which all agents have one goal, but for each agent it is characterized by its payoff (utility) function.

### Individually optimal allocations

The problem of collective decision-making in the following statement is being observed:

$$\begin{aligned} u_i(x) &\rightarrow \max, i \in N = \{\overline{1, n}\}, n \geq 2, \\ x \in X &= \prod_{i=1}^n X_i \end{aligned} \quad (1)$$

where  $u_i$  – a scalar utility function of  $i$ -th agent;  $X_i$  – set of its strategies;  $x = (x_1, \dots, x_n)$  – a situation the decision-making.

The principle of individual optimality is based on the following relation of domination in Nash sense.

*Definition 1.* Situation  $y = (y_i, x_{N \setminus i})$  is in relation of strong domination in Nash sense for agent  $i \in N$ , if

$$u_j(y_i, x_{N \setminus i}) > u_j(x), \forall j \in N. \quad (2)$$

Note that in (2) the effect of changes in strategy  $i$ -th player (from  $x_i$  to  $y_i$ ) to the values of utility functions of all players (in particular,  $i$ -th, which determines the Nash equilibrium) is analyzed.

*Definition 2.* The situation  $x^*$  is called weak individually-optimal equilibrium, if there is no such player  $i \in N$  and situation  $x \in X$  that would strongly dominate  $x^*$  in Nash sense.

Application of weak individually-optimal equilibrium is motivated by next scenario. Players enter into an optional agreement (the players listen to scientific advice of experts) that they would adhere to the situation  $x^*$ . If and only if the base of agreement is a weak individually optimal strategy, changes of  $i$ -th agent ( $i \in N$ ) strategy  $x_i^*$  that is agreed with the other players to any other, will always lead to a situation which would not be better for at least one player. As this situation cannot be the best for all players at the same time, then it is a compromise.

In [Maschenko, 2009] necessary and sufficient conditions for individual optimality, criterion of "stability" of the situation, the principles of choice of individually-optimal equilibrium are observed.

We assume that the payoff function of all the players are limited to a variety of situations  $X$  and denote  $S = \sum_{j \in N} \sup_{x \in X} u_j(x) < \infty$  - the sum of their upper limits. Consider for each player  $i \in N$  a set of parameters

$$M_i = \{\mu_i = (\mu_i^j)_{j \in N} \mid \sum_{j \in N} \mu_i^j = 1; \mu_i^j \geq 0, j \in N\}.$$

Necessary and sufficient conditions for individual optimality are established in the next theorem.

**Theorem 1.** If the situation  $x^*$  is individually-optimal, utility functions  $u_j(x^*) > 0, j \in N$ , then there are vectors of parameters  $\mu_i \in M_i, i \in N$ , in particular with the components:

$$\bar{\mu}_i^j = u_j(x^*) / S - \sum_{k \in N} u_k(x^*) / S n, j \in N, \quad (3)$$

that for  $\forall x_j \in X_j, i \in N$  the next inequalities are true:

$$\min_{j \in N} (u_j(x_i, x_{N \setminus i}^*) - S \mu_i^j) \leq \min_{j \in N} (u_j(x^*) - S \mu_i^j), \quad (4)$$

Any solution  $x^*$  of inequalities (4) for given  $\mu_i \in M_i, i \in N$  is weak individually-optimal equilibrium.

It should be noted that the parameters  $\mu_i \in M_i$  allow the player to express their preference on the set of payoff functions of all players  $u_j(x), j \in N$ . For example, if he considers that the function with index  $j_1 \in N$  is more important for him than the one with index  $j_2 \in N$ , he should choose  $\mu_i^{j_1} > \mu_i^{j_2}$ . By varying the parameters  $\mu_i \in M_i, i \in N$  you can find those or other equilibrium, by solving the inequality (4). On the other hand, each

individually-optimal equilibrium is characterized by some set of parameters  $\mu_i \in M_i$ ,  $i \in N$ , and, accordingly, the preference on the set of payoff functions of all players.

It should also be noted that the parameters  $\mu_i^j$  can be given some game sense. Suppose the situation  $x^*$  is weak individually-optimal equilibrium. Then according to Theorem 1, it is the solution of inequalities (4), at least when  $\mu_i^j = \bar{\mu}_i^j$ ,  $i, j \in N$  (according to (3)). It is easy to see that for each fixed  $i \in N$  the value of parameter  $\mu_i^j$  indicates the desired value of the payoff function  $u_j(x)$ ,  $j \in N$  in compromise  $x^*$ , according to the player  $i \in N$ . Then and only then it will not profitable for player  $i \in N$  to deviate from situation  $x^*$ .

Theorem 1 also shows that the situation  $x^*$ , which is an individually-optimal equilibrium of the original problem (1) for fixed values of the vectors of parameters  $\mu_i \in M_i$ ,  $i \in N$ , that characterize the preferences of the players on the set of criteria, is stable for any player  $i \in N$  on the function  $v_i(x, \mu_i) = \min_{j \in N} (u_j(x) - S\mu_i^j) + S/n$ , that characterizes the utility of the situation  $x \in X$  comparatively to changing strategy  $x_i^*$  to any other. Thus, individually-optimal equilibrium  $x^*$  is a Nash equilibrium in the game, which has a normal form  $(X_i, v_j(x, \mu_i); i \in N)$ .

Questions arise, whether a player who is in individually-optimal equilibrium will be tempted to change its strategy as well as his preference on the set of payoff functions of other players? If so, can such a situation be considered the foundation of a stable agreement between the players? If not, then which of the individually-optimal equilibrium and under which players' preferences are stable in this sense?

**Definition 3.** Set, which consists of the situation  $\hat{x} \in X$  and the parameter vector  $\hat{\mu} = (\hat{\mu}_i)_{i \in N} \in M = \prod_{i \in N} M_i$

will be called the equilibrium in the preferences of players, if  $\forall x_i \in X_i \forall \mu_i = (\mu_i^j)_{j \in N} \in M_i \forall i \in N$   
 $\min_{j \in N} (u_j(\hat{x}) - S\hat{\mu}_i^j) \geq \min_{j \in N} (u_j(x_i, \hat{x}_{N \setminus i}) - S\mu_i^j)$ . Denote  $RE$  - set of equilibria in the preferences of players.

Thus, in equilibrium in the preferences of players  $(\hat{x}, \hat{\mu})$  it's not profitable to any player  $i \in N$  to change both its strategy  $\hat{x}_i$  and the parameter vector  $\hat{\mu}_i$ , which characterizes his preference on the set of payoff functions of other players.

**Theorem 2.** Set  $(\hat{x}, \hat{\mu})$  is equilibrium in preference of players if and only if situation  $\hat{x}$  satisfies the following inequalities:

$$\sum_{j \in N} u_j(\hat{x}) \geq \sum_{j \in N} u_j(x_i, \hat{x}_{N \setminus i}), \quad \forall x_i \in X_i, \quad i \in N. \quad (5)$$

If the set  $(\hat{x}, \hat{\mu})$  is an equilibrium in the preferences of players, the situation  $\hat{x}$  is weak individually optimal equilibrium, and the components  $\hat{\mu} = (\hat{\mu}_i)_{i \in N}$  are calculated according to (3).

1 \ 2	L <sub>2</sub>	H <sub>2</sub>
L <sub>1</sub>	A 8,5; 8,5	B -1,5; -1,5
H <sub>1</sub>	C 1,5; 1,5	D -0,5; -0,5

A particular equilibria that maximize the overall utility function of players can be identified among the set of equilibria in the preferences of players.

*Definition 4.* The equilibrium in the preferences of players  $(\hat{x}, \hat{\mu}) \in RE$  will be called optimal if it maximizes the overall utility function of players, i.e.  $(\hat{x}, \hat{\mu}) \in \text{Arg max} \{ \xi(x) | (x, \mu) \in RE \}$ .

Conditions for optimality of equilibria in the preferences are set in the following theorem.

**Theorem 3.** Set  $(\hat{x}, \hat{\mu})$  is optimal equilibrium in the preferences of players if and only if situation  $\hat{x}$  satisfies the following conditions:  $\sum_{i \in N} u_i(\hat{x}) \geq \sum_{i \in N} u_i(x)$ ,  $\forall x \in X$ , and components of vector  $\hat{\mu} = (\hat{\mu}_i)_{i \in N}$  are calculated by

formula (3). Situation  $\hat{x}$ , which constitutes optimal equilibrium in the preferences of agents is Pareto-optimal situation of (1).

In particular, situation A will be optimal equilibrium in the preferences of agents for the game (Table 3). The vector of parameters that characterizes the preference of players, according to (3) will be the followed:  $\mu_1 = \mu_2 = (21 / 36, 15 / 36)$ . As a result, the situation A will become Nash equilibrium in game (Table 4), where the payoff of each player will be its utility function. Thus, taking in count the interests of each other will bring in situation A 9 units of additional profit to the first player and 6 to the second, in comparison with situation D (Table3). It should be noted that the "mechanisms" of taking in count the interests of one player by another are widely known and have long been used in practice (in particular, shares, securities, etc.). Therefore, this theory reveals just another aspect of their application for solving conflicts.

Note, that in [Voloshin, 2010] considered a somewhat different interpretation of the principle of "mutual consideration" of interests. It is offered to make the effective situation A (Table 3) a stable one (Nash equilibrium) by changing the initial matrix of the game in order to reallocate the players' payoffs. It requires a certain level of cooperation – at least an offer to "share gains". The proposed intuitive-understandable procedure requires a formal justification.

In this paper, constructive arrangements "taking into account the interests of each other" (on the basis of the above theory), which does not require co-operation of players, are proposed.

---

## Conclusion

The paper analyzes the principles of ineffective implementation of the Kyoto Protocol. Concepts which should be base for distribution of emission quotas mechanisms are proposed. It is alleged that the most appropriate principle of choosing the particular situation of collective decision in the allocation of emission rights is the principle of individual optimality, under which each party seeks to "compromise for the sake of conflict", choosing their strategies individually (noncooperative), but taking into account the interests of other participants.

---

## Acknowledgements

The paper is published with financial support by the project ITHEA XXI of the Institute of Information Theories and Applications FOI ITHEA and Consortium FOI Bulgaria ([www.itea.org](http://www.itea.org), [www.foibg.com](http://www.foibg.com)).

---

**Bibliography**


---

- [Samuelson, 2008] Samuelson P., Nordhaus V. Economics.–M.: Williams, 2008.–1360c. (In Russian)
- [Voloshin, 2006] Voloshin O.F., Maschenko S.O. Decision Making Theory.- K.: VTC "Kyiv University", 2006.– 304 c. (In Ukrainian)
- [Voloshin, 2009] Voloshin A., Goritsyna I. Mechanisms of emission quotas distribution under the Kyoto Protocol ";. In: International Book Series" Information Science and Computing ", 2009.-V. 3 / 2009, N10. - P. 175 -181 (In Russian)
- [Maschenko, 2009] Maschenko S.O. Individually optimal equilibria of noncooperative games in the relations of preference // Cybernetics and Systems Analysis, 2009, № 1 .- P.171-179. (In Russian)
- [Maschenko, 2009] Maschenko S.O. Equilibria stable by preferences in noncooperative single-purpose games // Kyiv University Bulletin. Series: math and physics. Science. - 2009. - № 3. - S. 152-157. (In Ukrainian)
- [Harsanyi, 2009] Harsanyi J., Zelten R. General theory of equilibria choice in games.-S.-Pb.: Economic School, 2001.-206 pp. (In Russian)
- [Voloshin, 2009] A. Voloshin, I. Goritsyna, S. Maschenko. Methodological principles of quotas allocation for emissions of greenhouse gases; In: "Natural and Artificial Intelligence", ITHEA, Sofia, 2010. - P. 85-93. (In Russian)
- 

**Information about authors**


---



**Olexij Voloshin** – Professor, Taras Shevchenko National University, faculty of cybernetics. Kyiv, Ukraine. E-mail: [ovoloshin@unicyb.kiev.ua](mailto:ovoloshin@unicyb.kiev.ua)

*Scientific Interests: decision making, systems of decision making support, mathematical economics, expert systems, e-learning*



**Sergij Maschenko** – Associate Professor, Taras Shevchenko National University, faculty of cybernetics. Kyiv, Ukraine. E-mail: : [msomail@yandexl.ru](mailto:msomail@yandexl.ru)

*Scientific Interests: optimization methods, decision making, systems of decision making support, game theory*