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ANALOGIES BETWEEN TEXTS: MATHEMATICAL MODELS AND APPLICATIONS IN COMPUTER-ASSISTED KNOWLEDGE TESTING

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***Abstract**: The origin for the classification of analogy is a comparison procedure we use to make a conclusion about similarity between the source and the target. Whenever the comparison procedure is not clarified, the analogy remains ambiguous. But if this procedure is formalized the analogy may allow to formulate and check conditions of its (still partial, but often very high) confidence. Below I discuss some ideas, which grounded certain algorithms allowed to calculate different kinds of similarity between texts, in particular of natural languages. The literal similarity of two words, as well as the lexical similarity of two given texts, can be estimated using so called "Theory of Finite Sequences Similarity". The structural similarity can be measured by fixing certain name groups in the source text and check the cohesion (proximity) of names belonging to corresponding groups in the target. Special logical formalism called "The Language of Ternary Description" can provide good templates for source text structuring when comparing texts of natural languages. It was demonstrated statistically that algorithms proposed for texts' analogy estimation provide "practically trustworthy" conclusions in the knowledge testing area. I argue also that high degree of confidence for that type of analogy is connected with the background of analogy (the notion which was discussed by G. Polya) though that background need not to be evidently formalized.*

***Keywords**: analogy, texts' similarity, knowledge testing, free answers*

***ACM Classification Keywords**: I.2.6 Artificial Intelligence – Learning – Analogies; K.3.1 Computers and Education - Computer Uses in Education*

Introduction

I assume the following general definition of analogical inference [Leonenko, 2008a]:

$L:\ \approx_{Cp}(a,b)\in\Omega,\ \mathrm{P}(a)\ \vdash\ \mathrm{Q}(b)$

where *a* is the source of analogy; *b* is the target; *L* is a language using for representing *a* and *b*; *Cp* is a procedure (algorithm) of comparison that estimates the degree of their similarity; Ω is a set of similarity values; P and Q are *L*–descriptions which characterize *a* and *b*; $\approx_{Cp}(a,b)\in\Omega$ means that similarity degree of *a* and *b* belongs to Ω. The procedure *Cp* serves as a basis for analogy types' classification.

Let the source *a* be a correct answer for test question, and let the target *b* be a variant of answer coming from the student. (P,Q)-pair corresponds to the information transferred from the source to the target, here it will be the correctness or admissibility of the answer (which is reflected by the "student's mark"). The restriction comes from computer is that both source and target must be represented as symbol sequences, or "texts" (if the answer is entered in non-textual form, the computer program will still deal with some symbolic representation of it). Analogical inferences take place in all cases when the student is allowed to use free form of the answer. Thus multiple choice is not in the case. But for "open" answers different comparison procedures *Cp* can be accepted for different presupposed structures of source and target objects.

For instance, if the answer is expected to be a set of numbers and the student's mark is calculated as a measure of intersection of this set and a "correct" one, then we obtain, factually, the traditional type of analogy – Aristotelian "paradeigma" [Uyemov, 1970]. But we meet with more complicated analogy types if take into consideration: 1) the differences in the used notions of numbers' identity (do we treat as identical real and integer numbers of the same value; or real numbers that are equal up to some accuracy; etc.); 2) the order in the two compared sets of numbers; 3) the condition of compactness for some subsets in the compared sets; and so on.

The other analogies appear when a computer program needs to recognize a mathematical expression in student's answer. Here the type of analogy depends on: 1) the kind of a given expression (e.g.: function, equation, identity, etc.); 2) the mode of expression's representation (e.g.: in some algorithmic language, or with given graphical primitives); 3) the equivalence class which is presupposed for the given kind of expressions – it determines cases when two different expressions must be considered as equivalent (here "equivalence" is the mode of interpreting "similarity"). For example, if the student's answer is a function represented as $b(x)$, we may assume that it can be accepted as correct if it coincides with the sample function $a(x)$ for one (or two, or larger, but finite number) argument value x_0: $b(x_0) = a(x_0)$.

Analogies between "texts" composed of "words" organized into "groups"

The most interesting are analogies using to recognize answers represented in natural languages (and also mixed answers, when student is allowed to use natural language together with mathematical expressions or other "artificial words" to express the answer). Here the assuming concept of "an object of *L* language" is "a text consisted of words". Let us assume the following terminology. Two words are called "similar" if there exists such common subsequence of their symbols which has "appropriate" length. Two texts are similar when contain many similar words. In spite of the simplicity of these initial concepts the mathematical model for that kind of analogy had required proving some nontrivial theorems. The most interesting is the fact that it provides very good results in real computer assisted testing processes. Answers that require natural language can be estimated by computer with necessary accuracy, and nonessential errors in answer can be disregarded.

Let me present one example. Imagine that we need to compare the following two sentences:

Newton and Leibniz were inventors of the mathematical analysis (**S**)

***The** math. **analisys was Newton**'s **and Leibni**tz's e**nvent**ion* (**T1**)

Let **S** be an "etalon" or "standard" answer for the question

What was the most important achievement in mathematics at the turn of the 17-18 c., and who was its author ? (**Q**)

Q is posed to the student in the computer assisted knowledge testing session; and the phase **T1** is a factual answer coming from the student. **T1** contains several mistakes and changes in words (underlined). The order of words in **T1** differs from that in **S**. But it is clear that **T1** is a correct answer to the question **Q**.

We see that there exists certain similarities between some words of **S** and **T1**, e.g.:

{***inventors*** , *e**nvent**ion* } or {***analysis*** , ***anali**s**y**s*},

– two words in each pair are "analogical".

One can say that "analogical" is not appropriate word here. But let me consider any word in the phrase as "an object"="a complex of attributes", where each "attribute" is an *occurrence* of a symbol in that word (i.e. a pair <symbol, position>). The position is treated as not a numerical index, but rather a relation between the set of previous and the set of next occurrences. Two words are considered as having "the same attribute" if for a given occurrence of their common symbol two sequences of letters that stand before/after that symbol are identical in each word. Furthermore, two words are considered as having "similar attributes" if the mentioned sequences "slightly differ" one from another. Finally, two words are considered to be the more similar the more "similar attributes" they have. Thus we come to something very closed to the traditional analogy of Aristotle (paradeigma), though our notion of "attribute" includes references to other "attributes", and so factually we have a mix of paradeigma and proportional analogy.

Specializing the above ideas, there can be introduced measure function F(a,b) that estimates analogy between two words a and b [Leonenko & Poddubny, 1996]. It can be shown that this measure produces the correspondent notion of the "distance" between words:

Theorem 1. The function 1-F(a,b) forms a metric space in the set of words.

We can make further assumptions to precise the notion of words' similarity in the considered case. E.g., we can postulate that some letters in a word can be more "essential" than others, and assign certain "weight" to each symbol in a word:

i	n	v	e	n	t	o	r	s
1	1	1	1	1	1	0	0	0

There can be proposed measure function G(*a*,*b*,W) to calculate "the degree of analogy" treating as weighted similarity of words [Leonenko, 2002]. Applying it, we will obtain, e.g., the value **0.833** for the pair of words

{***inventors*** , <u>e</u>***nventi***<u>on</u> }

The third parameter W in G(*a*,*b*,W) reflects the "expecting potential weight" of those letters of the target *b* which are not present in the source *a* and therefore can't be weighted directly. Thus G, unlike F, is not a symmetric function with respect to *a* and *b*. Nevertheless, G has the following "natural" properties:

Theorem 2. (1) if $W_1 \leq W_2$ then $G(a,b,W_2) \leq G(a,b,W_1)$;

(2) $0 \leq G(a,b,W) \leq 1$ for every *a*,*b*,W;

(3) if W>0 and G(*a*,*b*,W)=1 then *b* coincides with *a* up to some letters that have zero weights;

Passing from words to phrases, we see that there exists a similarity between "source" phrase **S** and "target" phrase **T1**: they are "analogical" in the sense that have many "analogical" words; – just like two words that have many common symbols, but with the addition that the *order* of words in phrases is *less rigid* than the order of letters in words. There was introduced the measure function $G_U(a,b,W)$ that calculates weighted similarity *fully ignoring* the order. So if we assign, e.g., the following weights to words of **S**:

Newton	***and***	***Leibniz***	***were***	***inventors***	***of***	***the***	***mathematical***	***analysis***
25	0	25	0	10	0	0	10	30

then G_U(**S**,**T1**,W) gets the value **1.00**, thus with respect to G_U **S** and **T1** appear as "fully analogical" phrases. There exist some "natural" properties and relations between G(*a*,*b*,W) and $G_U(a,b,W)$ measures:

Theorem 3. (1) if W>0 and $G_U(a,b,W)=1$ then *b* is a permutation of those sub-objects of *a* that have non-zero weights;

(2) $G(a,b,W) \leq G_U(a,b,W)$ for every *a*, *b* and W.

The next step towards more accurate estimation of analogy between phrases can take into account certain structures in phrase. Consider, e.g., the following combination of words:

The mathematics of Newton invents Leibniz's analysis (**T2**)

Though **T2** includes analogs of *all* non-zero weighted words of the "source" phrase **S**, **T2** does not look like a correct answer to the question **Q**. So we need to account that shuffling of words in **T2** affects in decreasing its similarity to **S**. There can be proposed the measure for "the cohesion" of given group of words in the phrase [Leonenko, 2007]. It is obvious, that in both **S** and **T1** each of the two following groups – {***Newton***, ***Leibniz***} and {***mathematical***, ***analysis***} – are cohesive, while in **T2** they are not.

Let *a* and *b* be some words belonging to the same group ***g***. In order to formalize the notion of "cohesive group" we may start with the definition of the "measure of disconnection" of words *a* and *b* in a given text **T**. It is defined as a number $\rho(a,b)$ of those words, which occur in **T** between *a и b* and not belong to ***g***. It is easy to see that $\rho(a,b)$ is a distance function on the subset of those elements of ***g*** which occur in **T**. Let **T** contain exactly ν occurrences of group ***g***'s elements, and let $\nu \geq 1$. Assign the indexes $1,\ldots,\nu$ to these occurrences preserving their order in **T**; and designate the obtained set $\{c_1,\ldots,c_\nu\}$ as ***g***[**T**]. In the case of $\nu>1$ I will call "the disconnection matrix" of the set ***g***[**T**] the $\nu\times\nu$-dimensional matrix **M**, whose elements $\mathbf{M}_{q,r}$ are equal to $\rho(c_q, c_r)$ in the sequence **T**.

"The average disconnection" μ(***g***[**T**]) of the set ***g***[**T**] is defined as zero when ν**=**1, and otherwise ($\nu>1$) as:

$$\mu(\boldsymbol{g}[\mathbf{T}]) = \frac{\sum_{q=1}^{\nu}\sum_{r=1}^{\nu} \mathrm{M}q,r}{\nu(\nu-1)},$$

where **M** is the disconnection matrix of ***g***[**T**].

"The spectrum of the disconnection" of the set ***g***[**T**] is defined as the sequence of natural numbers L(***g***[**T**]) = <L_1, … , $L_{\nu-1}$>, where $L_k = \rho(c_k, c_{k+1})$, $k \in [1, \nu-1]$. The following theorem shows that it is easy to calculate μ(***g***[**T**]) using the spectrum of the disconnection:

Theorem 4. $\mu(\boldsymbol{g}[\mathbf{T}]) = \frac{2}{\nu(\nu-1)} \sum_{j=1}^{\nu-1} j(\nu-j)L_j$.

Finally, let me introduce the notion of so called "φ –cohesion". Let φ be an arbitrary function with the following properties:

0) $\varphi(x)$ is determined for every $x \geq 0$;

1) $\varphi(0) = 1$;

2) $\varphi(x)$ is non-increasing for $x \to \infty$.

Then φ–cohesion of group ***g*** elements in the text **T** is defined as φ(μ(***g***[**T**])).

The φ–cohesion can be used as the specific measure of similarity for those subsequences (groups) of words in the sentence, which are somehow "clogged" with alien, extraneous words. This type of similarity may be resembled to the notion of "recognizing of the sub-phrase". The condition 1), which must be satisfied by φ , means that if there are no extraneous words between words from ***g*** in **T**, then the corresponding sub-phrase can be "fully recognized". Otherwise the degree of its recognition can be (not obligatory) less than 1.

The concrete form of the function φ can be chosen under the accepted hypothesis of how the ability to recognize the given sub-phrase decreases when this sub-phrase becomes "clogged". If one accepts that the only one extraneous word strongly affects the recognition, then it may be suitable to choose $\varphi = e^{-x}$. But if it is accepted that a sub-phrase can be "half-recognized" independently of the number of words that "clog" it, one can chose φ =(1+x)/(1+2x), which has the limit 1/2 when $x \to \infty$.

Let us look how the above measure of φ–cohesion works being applied to the English sentences considered above. We must first divide the standard sentence **S** into the nonintersecting sets of words. Let

$\boldsymbol{g}_1$ be the set of words, which are analogous (i.e. similar in respect of common letters, their weights and order) to one of the two words: ***Newton*** or ***Leibniz*** on the appropriate similarity levels;

$\boldsymbol{g}_2$ be the set of words, which are analogous to one of the words: ***inventors***, ***founders***, ***devisers***, and so on;

,

Now when we will compare the standard phrase **S** to an arbitrary phrase **T**, we will form sets $\boldsymbol{g}_i$[**T**] as $\boldsymbol{g}_i[\mathbf{T}] = \mathbf{T} \cap \boldsymbol{g}_i$, $i \in [1, 3]$.

If the order of words in phrases **S** and **T** is *not* taken into consideration, the similarity between these two phrases is proportional to the total weight of their common words. Let us now assume that the total weight of the group $\boldsymbol{g}_i$[**T**] *must be decreased by multiplying it by the φ–cohesion* of $\boldsymbol{g}_i$[**T**] in **T**.

Then, if we will take $\varphi(x) = 1/(1+x)$ to calculate the φ–cohesion, we will obtain the following results:

	The value of the weighted similarity to **S** when φ–cohesion is ignored	The value of the weighted similarity to **S** when φ–cohesion is taken into account
T1	1.00	1.00
T2	1.00	0.45

Thus using the measure of cohesion *together* with the measure of weighted similarity, we decrease the value for the analogy between **S** and **T2**, preserving it for **S** and **T1**.

The last problem I will discuss in this section is the problem of fixing the nonintersecting groups of words in a given "standard" answer. If we do not leave this problem to the intuition of a person who prepares the testing question, we may turn either to the linguistics of a given natural language or to some logical analysis of a phrase. The last way presupposes some formal logical apparatus. It is well-known that different logical calculi have different abilities in reflecting characteristics of natural phrases. Let me notice the logical calculus called "The Language of Ternary Description" (LTD), elaborated by A. Uyemov [Uyemov, 1995]. The "source" sentence **S** considered above has the following LTD-model:

i (*{{ *N*, *L* } • [(*a*)*m*] }),

where **i** denotes "invention"; **N** – "Newton"; **L** – "Leibniz"; **a** – "analysis"; **m** – "mathematical".

There was proposed the hypothesis that cohesive groups in natural language's sentence correspond to the nonintersecting 1st level sub-formulas of the LTD-model of that sentence [Leonenko, 2008b]. For the above sentence **S** these will be sub-formulas ***i***, **{*N*, *L*}** and **[(*a*)*m*]**. Note that parentheses and square brackets in LTD denote special operations of predication and "reification", that's why ***i*** is a separate sub-formula in ***i* (*{{ *N*, *L* } • [(*a*)*m*] })**. (The function of arrangement is played by the braces.)

When the hypothesis stated above is used in the processes of calculating the φ–cohesion, it is assumed, roughly speaking, that words of the given LTD-vicinity must be closed enough one to another in the natural sentence. This assumption was confirmed by testing different natural sentences (mainly from Ukrainian and Russian languages). In particular, we can see that it works for the above English sentences **S**, **T1** and **T2**.

It can be showed that algorithms which estimate different types of texts' similarities using the above measure functions have polynomial computational complexity.

The described methods of evaluating analogies between sentences were checked on the large amount of texts in computer assisted knowledge testing sessions. They showed very good practical results. In fact, it was statistically proved that these measures for analogies give practically same conclusions about student's attainments to those obtained by manually checking written tests allowing "open" answers of medium length [Baranov, 2004].

Overcoming the "gaps of inference" to get trustworthy conclusions of analogy

Though statistical confirmation is important, there still remains the following problem. Let a computer had estimated *concrete* student's answer **T** to some *concrete* question as highly similar to an "standard" answer. What must be the degree of our confidence in the correctness of the answer **T**?

This problem is connected with the general problem of the validity of analogical inferences. I will discuss some aspects of it in this last section.

At first, I want to draw attention to the following fact. The notion of "an attribute" in examples of analogies between texts given above had become very specific. As far as I acquainted with different theoretical and practical approaches to analogy (e.g., in probabilistic logic, structure mapping, expert systems, etc. – see [Niiniluoto, 1988], [Gentner & Kurtz, 2006], [Kokinov & French, 2003], [Spanoudakis & Constantopoulos, 1996]), the deep specifications of the assumed notions of "an object", "an attribute of object", "a relation between objects" and "a similarity between objects" appear as necessary conditions in all the cases when the conclusion of analogy has high degree of certitude. In most of these cases one can make fully transparent his claim that two objects are "similar", and usually can present the algorithm to compare the source and the target and evaluate their similarity.

The so-called "validity principles of analogy" [Uyemov, 1971] in situations when the comparison procedure can be make precise transform to special rules (sub-algorithms) of comparison. Thus, the traditional rule of paradeigma "take the most important attributes" transformed in examples in previous section into "select analogs with the high total weight".

When comparison procedure can't be described precisely the validity principles become fuzzy, like "take as many properties as possible" in classic paradeigma. The last principle can even be unacceptable in some precise types of paradeigma (in previous examples another principle works: "*do not* take properties of zero weight"). If the rules for comparing objects are fuzzy, we can't be sure that two given objects are "really" similar. This is our main reason to say that the conclusion of analogy is "only questionable".

So I insist that in general case the existence of an algorithm that can *evaluate* the *degree* of similarity between the source and the target is the *necessary condition* for analogy to produce confident results.

But what about the *sufficient* conditions? When the comparison procedure is precise and gives high degree of similarity, we often rightfully trust the conclusion of analogy. Why can we do this?

Sometimes it happens because the conclusion is deductively entailed from the similarity of the source and the target (like in strict isomorphism). But in most cases there exists a "gap of deduction" in making analogical conclusions. I now discuss one case when we can ignore this gap.

Let me return to the area of computer assisted knowledge testing. Consider the following kind of analogy used to evaluate the answer that is a mathematical function, e.g. $f(x)=0.35 sin 2x$. It is obvious that if the student will enter any formula equivalent to $f(x)$, like $g(x)=0.7 sinx\, cosx$, his answer will be correct. The usual way for computer to check such answers is to compare the values of $f(x)$ and $g(x)$ for one (or two, or larger, but *finite* number) argument value x_0: $f(x_0) = g(x_0)$.

If we fix the appropriate value for x_0 (like $x_0=0.314$ for the above example), we can be *absolutely sure* that the coincidence of the student's and the "standard" numerical results guarantee the correctness of his answer. But this kind of logical inference can be treated as a paradeigma with the *single* attribute of object (or, if you wish, as an enumerative induction with the single trial). Traditional "validity principles" for paradeigma (or induction) do not recommend accepting such conclusions. In fact, there exists an infinity of functions that coincides with $f(x_0)$ on the argument $x_0=0.314$.

I think that high degree of confidence we tend to assume for that type of analogy is connected with the *background* of analogy (see [Polya, 1954]). The student (even if he knows or guesses the method of comparing his and standard answer) needs to know the concrete value of x_0 to "deceive" the computer. This is almost impossible in the considered situation (i.e. on the given background of analogy).

It can be said that the precision of the comparison procedure of the source and the target of analogy must be bring up to the extent when the background of analogy will give us the possibility to overcome the "gap of deduction". E.g., in the case of comparing textual student's answers we come to the "1-st approximation" of such a background when guarantee that "most of *essential* words of standard answer *somehow* occur in the student's one". The next step may be done when we can check if "most groups of names that *are cohesive* in the standard answer *are cohesive* in the student's one", and so on.

In other words, the sufficient conditions for the validity of analogy are related to those – maybe, extra-logical – circumstances (background), which allow to assume that when taking the target with high similarity to the source we can (under this background) perform the transfer of information.

The practical examples discussed above show that in some cases we may fix the background of analogy to ensure trustworthy conclusions, though this background includes extra-logical assumptions.

Conclusion

There exist different aspects in which two given texts of given natural or artificial language can be treated as being "analogical". If we consider "text" as a "sequence of phrases", and "phrase" as a "sequence of words", then the Theory of Finite Sequences Similarity gives us tools to estimate degrees of literal similarity between words and lexical similarity between phrases or texts. We can also use logical calculi appropriate to fix "cohesive" structures in natural languages' texts, and then apply mathematical measures to check the cohesion of a given name group in text being the target of analogy. It gives the possibility to estimate the similarity of texts' structures. Taking together, these methods can make a contribution, in particular, to the problem of "free" answers in computer-assisted knowledge testing. Namely, the algorithms discussed above are able to disregard nonessential lexical and structural changes in a given “free” answer. As a result, estimations of “free” answers produced by computer do not differ from those of humans. It was confirmed by statistical experiments, but it also can be justified by taking into account the specific background of analogies in automatic knowledge testing area.

Bibliography

[Baranov, 2004] V. Yu. Baranov. Computer-assisted testing in informatics: the results of pedagogical experiment in Odessa National Academy of Telecommunications // Theory and methods of education in mathematics, physics, informatics. Issue 4, Vol.3. – Krivyi Rig, 2004, p.6-12. (In Ukrainian).

[Gentner & Kurtz, 2006] D. Gentner, K.J. Kurtz. Relations, Objects, and the Composition of Analogies // Cognitive Science, Vol.30, 2006, p.609–642.

[Kokinov & French, 2003] B. Kokinov, R.M. French. Computational Models of Analogy-making // Encyclopedia of Cognitive Science. Vol. 1. London: Nature Publ. Group, 2003, p.113-118.

[Leonenko, 2002] L. Leonenko. Analogical inferences in computer assisted knowledge testing systems // 6th Multi-Conference on Systemics, Cybernetics and Informatics (SCI 2002), Orlando, Florida, USA, 2002. Proceedings, Vol. XVIII, p.371-376.

[Leonenko, 2007] L. Leonenko. Algorithms for estimation analogies between texts and its applications in computer assisted knowledge testing // VII Intern. Conf. “Intellectual Analysis of Information”. Transactions. – Kyiv: Prosvita, 2007, p. 210-220. (In Russian)

[Leonenko, 2008a] L. Leonenko. Analogy and Abduction // Filosofska Dumka, 2008, No. 3, p.14-30. (In Ukrainian)

[Leonenko, 2008b] L. Leonenko. The Language of Ternary Description in Text’s Cohesion Estimations // VIII Intern. Conf. “Intellectual Analysis of Information”. Transactions. – Kyiv: Prosvita, 2008, p. 286-295. (In Russian)

[Leonenko & Poddubny, 1996] L. Leonenko. H. Poddubny. The Theory of Finite Sequences Similarity and its Applications to Image Recognition // Automatics and Telemechanics, 1996, No.8, p.119-131. (In Russian)

[Niiniluoto, 1988] I. Niiniluoto. Analogy and Similarity in Scientific Reasoning // Analogical Reasoning. – Synthese library, Vol.197, Kluwer Acad. Publishers, 1988, p.271-298.

[Polya, 1954] Polya G. Mathematics and Plausible Reasoning.– Princeton Univ.Press, NJ, 1954.

[Spanoudakis & Constantopoulos, 1996] G. Spanoudakis, P. Constantopoulos. Elaborating Analogies from Conceptual Models // International Journal of Intelligent Systems, 1996, Vol.11, No.11, p.917-974.

[Uyemov, 1970] A. Uyemov. Analogy in the Practice of Scientific Researches. – “Nauka” Press, Moscow, 1970. (in Russian)

[Uyemov, 1971] A. Uyemov. The logical foundations of modeling method. – Moscow: “Mysl” Press, 1971. (In Russian)

[Uyemov, 1995] A. Uyemov. The Language of Ternary Description as a Deviant Logic // Boletim da Sociedade Paranaense de Matematica. – 1995, Vol.15, No.1-2, p.25-35; 1997, Vol.17, No.1-2, p.71-81; 1998, Vol.18, No.1-2, p.173-190.

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