DYNAMIC SYSTEM QUALITY PROVIDING UNDER UNDETERMINED DISTURBANCES. ONE-DIMENSIONAL CASE

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Abstract: *One-dimensional dynamic system under impact of the undetermined disturbing influences is reviewed. The possibility to affect only the value of disturbances is conceded. An algorithm for system quality estimation and making decision about control aiming to provide required quality is proposed. Control algorithm for one-dimensional system of any order is developed.*

Keywords: *dynamic system quality; undetermined disturbances; condition estimation; resulting disturbance control; control algorithm; quality function.*

Introduction

Quality maintenance of the dynamic systems under random undefined disturbing influences does not have any unequivocal solution. Existing methods presuppose either complete a priori information about disturbances, or their constraints are known [Lin, Su, 2000], [Poliak, Sherbakov, 2002], [Nikiforov, 2003], [Hou, Muller, 1992], while regulators with dynamic disturbance compensators might have high dimensions [Liubchyk, 2007].

System Condition Estimation

Here we review a steady linear system with one input and one output. Input F and output α value correlation is set by the dynamic operator $\Phi(p)$

$$\Phi(p)\alpha = F , \tag{1}$$

where $p = \frac{d}{dt}$ – time differentiation operator.

α variable defines system functioning quality when under control, and drift from the needed quality (error) when under disturbances. Without problem contraction, it is possible to review linear system (1) behavior but only under disturbances.

Using system (1) weight function $w(t)$, it becomes possible to estimate input value $\alpha(t)$ under disturbances $F(t)$,

$$\alpha(t) = \int_0^t F(\tau)w(t-\tau)d\tau = \int_0^t \psi(\tau)d\tau , \tag{2}$$

$$\psi(\tau) = F(\tau)w(t-\tau) . \tag{3}$$

Proceeding from geometrical interpretation of expression (2), it is possible to estimate output value of the system by its square, described in time by the function $\psi(\tau)$ on the observation interval (0, t) (Fig.1).

On the other hand, if the system (1) is observable, we can estimate the disturbance F(t). If $\alpha(t)$ can be differentiated n times (n- system (1) operator p polynomial order (1)), its derivatives can be received after $\alpha(t)$ measuring.

In this way, to estimate system quality it is sufficient to have to have as time function $\psi(\tau)$ depending on the acting disturbances and system (1) dynamic features.

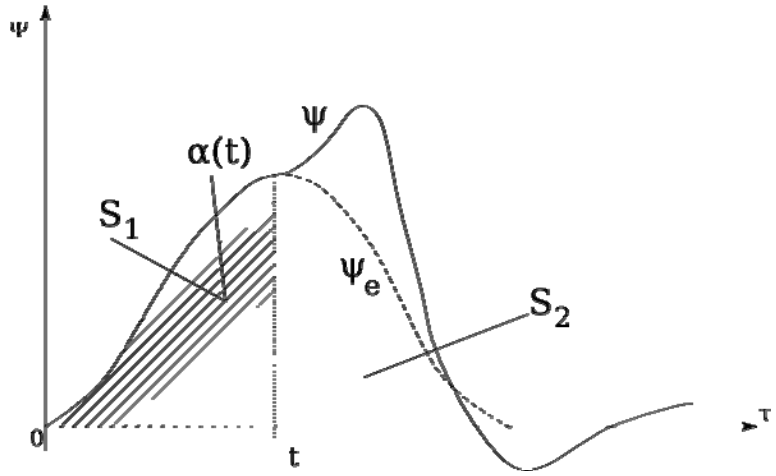

Figure 1. $\alpha(t)$ *estimation*

Making Decision about Starting Disturbance Control

Here we review system (1) quality providing as definition of its drift α (input value bias from the program value) in allowed ranges.

Using geometrical interpretation of (2) to provide needed system quality it is necessary and sufficient to provide function $\psi(\tau)$ (quality function) value in the range allowed (quality range). Its square S_Q does not exceed maximum value α_{al} of on the interval of constant-sign $\psi(\tau)$ function,

$$S_Q \le \alpha_p\,,\quad t \in (t_1, t_2)\,,\quad S_Q \in (S <> 0) \tag{4}$$

Quality range may be constructed as follows. Let us assume that system (1) is a control object in a feedback control system undergoing external disturbance F_{ext}. Then, $F(1)$ is a resulting disturbance, the aggregate of the external disturbance and feedback F_y action. If the system is steady, closed, and of the needed quality when under typical disturbances, resulting disturbance $F(1)$ has variable sign, both under constant-signed and variable signed external disturbances. Let us estimate (or measure) value on interval $t \in (0, t_k)$, where t_k - moment of time, when

$$\alpha(t_k) = \varepsilon\alpha_p, \quad 0 < \varepsilon < 1, \tag{5}$$

and quality function (3)

$$\psi(\tau) = F(\tau)w(t_k - \tau),$$
$$\tau \in (0, t_k) \qquad (6)$$

Range (2) S_1 for the function (6) will thus fill a part of quality range (4) S_Q .

With disturbance type undefined, let us set $\varepsilon = 0.5$. Then the second part S_2 of the quality range can be constructed as function (6) mapping relative to $t = t_k$ at the range $t > t_k$:

$$\psi(t) = \psi(2t_k - t) \,. \qquad (7)$$

Meanwhile it is necessary that

$$S_1 + S_2 \le S_Q \,, \quad S_1 = \alpha(t_k) \,, \quad S_2 = \alpha(t) - \alpha(t_k) \,, \qquad (8)$$

and $t = t_k$ is a moment of disturbance control start.

Disturbance Control Start

Let us define the possibility of controlling disturbance value when it acts over the control object (1) included in the closed system. After having implemented a closed control system, as shown in the Fig. 2, under certain constraints on control operator $\Phi_y(p)$ parameters,

$$\alpha = \frac{\Phi^{-1}(p)}{1 + K\Phi^{-1}(p)\Phi_y} F_{ext} = \frac{p}{KH(p)} F_{ext} = \frac{W(p)}{K} F_{ext} \,,$$

$$F = \frac{p\Phi(p)}{KH(p)} F_{ext} = \frac{W(p)\Phi(p)}{K} F_{ext} \,. \qquad (9)$$

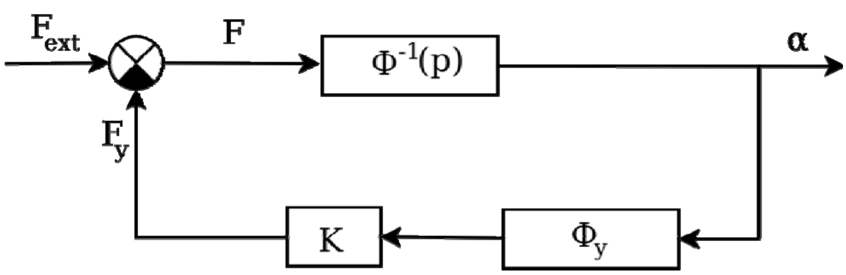

Figure 2. Control system. Example 1

Without constraints on the control operator we can get a similar result in the system with double feedback (Fig. 3), where operator w defines closed system (8):

$$\alpha = \frac{W(p)}{1+K} F_{ext} \,, \quad F = \frac{W(p)\Phi(p)}{1+K} F_{ext} \,. \qquad (10)$$

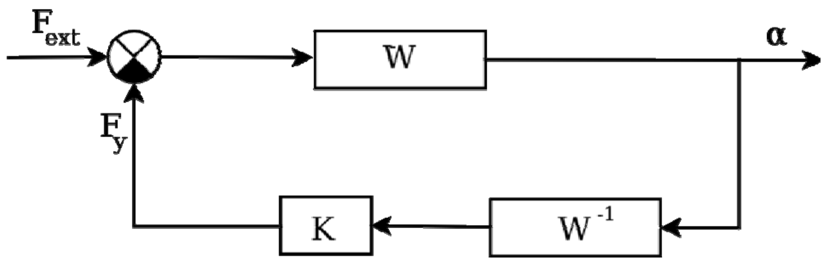

Figure 3. Control system. Example 2

Thus, in the control systems (9), (10) external disturbances are varied with the operator K by its value. In this way disturbance alteration remains unchanged both for the resulting disturbance and for the input value.

Let us define a control algorithm according to (7) and (8). Let us set quality value (7) as etalon.

$$\psi(2t_k - t) = \psi_e(t),\quad t > t_k,\quad \alpha > \alpha_e. \tag{11}$$

Let us name the difference between real α and etalon α value for the input

$$y = \alpha - \alpha_e = \int_0^t [\psi(\tau) - \psi_e(\tau)] d\tau,\quad |\alpha| > |\alpha_e|, \tag{12}$$

and use direct Liapunov method for control $K(t)$ definition in the case (10), or $K_*(t)$, $K(t) = 1 + K_*(t)$ for (9). Let us introduce Liapunov function $V = y^2$. Then it is necessary to provide

$$\frac{dV}{dt} = 2y\frac{dy}{dt} < 0, \tag{13}$$

where

$$y(t) = \int_0^t [\psi_k(\tau) - \psi_e(\tau)] d\tau,\quad \frac{dy}{dt} = \psi_k(t) - \psi_e(t),$$

$$\psi_k = \frac{\psi}{1+K} \qquad (\psi_k = \frac{\psi}{1+K_*}). \tag{14}$$

Condition (13) is met if

$$K(t) > \frac{\psi(t) - \psi_e(t)}{\psi_e(t)},\quad t > t_k,\quad |\alpha| > |\alpha_e|,\quad K(t) > 0. \tag{15}$$

For that operator K formation rule may be chosen as one of the following:

$$K(t) = \left[\frac{\psi(t)}{\psi_e(t)} - 1\right]\left[1 + sign(|\alpha| - |\alpha_e|)\right]\left[1 + sign(\psi - \psi_e)\right], \tag{16}$$

$$K(t) = \left[\frac{\psi(t)}{\psi_e(t)} - 1\right]\left[1 + sign(|\alpha| - |\alpha_e|)\right]\left[\frac{\alpha}{\alpha_p} - \varepsilon\right]^2\left[1 + sign(\psi - \psi_e)\right] =$$
$$= \frac{y}{\psi_e}\left[1 + sign\frac{dy}{dt}\right]\left[1 + sign(|\alpha| - |\alpha_e|)\right]\left[\frac{\alpha}{\alpha_p} - \varepsilon\right]^2. \tag{17}$$

Operator becomes undefined when $\psi_e = 0, y > 0$.

To avoid uncertainty in (16), (17) algorithms it is sufficient to put

$$K(t) = K(\psi_e \to 0_+, y > 0) = const \quad (18)$$

on the time interval starting from the moment (18) until the moment when $\psi(t) = 0$.

Conclusion

Undefined external disturbance control rule described is based on the estimation not the disturbance itself but dynamic quality function system features, depending on it. Such estimation allows to forecast the possible scale of quality function changes and to make a decision about control start. Control algorithm, developed on the basis of current and estimated quality function values, provides the necessary quality of the one-dimensional dynamic system.

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