Business Intelligence Systems

APPLICATION OF DISCRETE OPTIMIZATION IN SOLVING A PROBLEM OF MULTI-ITEM CAPACITATED LOT-SIZING WITH ECONOMIC OBJECTIVES

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Abstract: In this paper we study the problem of multi-item capacitated lot-sizing from the point of commercial enterprises. We consider profit as the main criteria. This dynamic problem belongs to the class of discrete optimization and contains boolean variables, algorithmic objective function, where various types of constraints such as analytical functions, algorithmic and simulation models can be used. We present model and direct search algorithm that consists of an intelligent iterative search and upper bound set construction, and allows finding exact solution in reasonable time. We carry out computational investigation and solve a real task with using developed computational tool to show the efficiency and practical application of the proposed model and algorithm.

Keywords: direct search; discrete optimization; upper bound; production planning; multi-item capacitated lot-sizing; profit maximization

ACM Classification Keywords: G.1.6 Optimization; J.0 Computer Applications

Introduction

The task of determining the maximum-profit production plan for lot-sizing problem is a very important issue for commercial enterpreises. The quality of the enterprise strategy highly depends on the successful solution of this problem.

Traditionally, the main criteria for the problem of lot-sizing was a criteria of minimizing the production costs to all phases of the product's life cycle [Absi, 2008], [Pochet, Wolsey, 2006], [Walser, 1999].

The paper [Beresnev, Gimadi, Dementyev, 1978] reviews various models of product lot-sizing optimization. An algorithm based on the branch-and-bound method was also proposed. The proposed mathematical models do not take into account the possibility of using the algorithmic objective function and constraints that help the model to reflect the object properties and behaviour more accurately.

Several types of stochastic models, that describe the problem of product lot-sizing optimization, were proposed in the paper [Antipenko, Katz, Petrushov, 1990]. This approach allows to find a local extremum of multiextremal objective function. Considered models take into account the costs of product customization according to specific consumer needs.

The paper [Kononenko, 1990] proposes models of the dynamic product lot-sizing optimization. The models contain single objective function and different types of constraints: analytical, algorithmic, and simulation models. The proposed algorithms allow to find an exact solution.

In the paper [Kononenko, Rogovoi, 2000] a mathematical model and non-Markov approach to multi-objective optimization for a dynamic product lot-sizing were developed. The proposed model minimizes costs in all phases of the product life cycle, and takes into account specific needs of the consumers. The model's constraints can be of various types such as analytical expressions, algorithms or simulation models. The method is based on minimax and direct search approaches.

The paper [Kononenko, Derevjanchenko, 1999] reviews an approach and a fuzzy constraint based model of dynamic product lot-sizing optimization. To enshure high adequacy of mathematical models the optimization-simulation approach [Tsvirkun, Akinfiev, Filippov, 1985] was proposed for mathematical-programming problems containing algorithmic objective function and constraints.

Use of profit as the main criteria for solving the lot-sizing problem can be beneficial for the commercial enterprises and make them more competitve in changing market conditions [Kononenko, Protasov, 2005],[Kononenko, Protasov, Protasova, 2005].

The purpose of this research is to develop a mathematical model and an approach to the solution of dynamic multi-item capacitated lot-sizing problem with using profit as the main criteria. The constraints of the model can be of various types: analytical, algorithmic or simulation models.

Description of the proposed mathematical model

Let's assume that product *i*, $i=\overline{1,m}$ is used to meet needs of consumer *j*, $j=\overline{1,n}$. *Pij* is the production quantity of product *i* that is required to fullfill the demand of consumer *j*. *T* is a period of production planning.

The mathematical model of multi-item capacitated lot-sizing problem uses profit maximization as the main criteria. This model is formulated below.

$$\sum_{t=1}^{T}\sum_{i=1}^{m}\left[Ц_i\left(\sum_{j=1}^{n}p_{ij}x_{ij}\right)\right]\times\left(\sum_{j\in\{j:]j/\chi[=t\}}p_{ij}x_{ij}\right)\alpha_t - \sum_{i=1}^{m}\left[w_i\left(\sum_{j=1}^{n}p_{ij}x_{ij}\right)\right]\alpha_{t_i^{'}} -$$

$$\sum_{t=1}^{T}\sum_{i=1}^{m}\left[v_i\left(\sum_{j=1}^{n}p_{ij}x_{ij}+p_{i,пред}\right)\right]\times\left(\sum_{j\in\{j:]j/\chi[=t\}}p_{ij}x_{ij}\right)\alpha_t - \sum_{j=1}^{n}\sum_{i=1}^{m}z_{ij}\alpha_{t_j}x_{ij}\to\max_{x_{ij}} \quad (1)$$

$$S_t = S_{t-1}\alpha_t/\alpha_{t-1}+K_t\alpha_t-\sum_{i\in\{i:t_i^{'}=i\}}w_i\left(\sum_{j=1}^{n}p_{ij}x_{ij}\right)\alpha_{t_i^{'}}, \quad (2)$$

$$S_t\geq 0,\ t=\overline{1,T},$$

$$b_t^{(h)}=\varphi\left(b_{t-1}^{(h)},x_{ij}\right), i=\overline{1,m},\ j\in\{j:]j/\chi[=t\}; \quad (3)$$

$$d_t^{(h)}=\sum_{j\in\{j:]j/\chi[=t\}}p_{hj}x_{hj}$$

$$b_t^{(h)}\geq d_t^{(h)}$$

$$\alpha_t^{(q)}=f\left(\alpha_{t-1}^{(q)},x_{ij}\right), i=\overline{1,m},\ j\in\{j:]j/\chi[=t\}; \quad (4)$$

$$\alpha_t^{(q)}=\begin{Bmatrix}\leq\\ \geq\end{Bmatrix}e_t^{(q)}\ \ \forall q\in Q, t=\overline{1,T};$$

$$x_{ij}\in\{0,1\}, i=\overline{1,m},\ j=\overline{1,n} \quad (5)$$

$$\sum_{i=1}^{m}x_{ij}=1,\ \ j\in U$$

$Ц_i\left(\sum_{j=1}^{n} p_{ij}x_{ij}\right)$ function is used as the price of product *i* and depends on the production volume.

$w_i\left(\sum_{j=1}^{n} p_{ij}x_{ij}\right)$ function defines the preproduction costs that include costs for scientific research, experimental development, production tooling, etc.

$v_i\left(\sum_{j=1}^{n} p_{ij}x_{ij}\right)$ function defines the production costs per unit of product *i* and depends on the production volume.

z_{ij} defines the costs for transportation, adjustments, and possible setup costs that are required for the prouct *i* to meet the needs of the consumer *j*.

Φ is a set of products that were previously developed and do not need preproduction.

$t_i^{'}$ is the year when product *i* was developed.

$t_i^{'} = \underline{\min}_{j=1,n} T_i$, если $i \notin \Phi;\ T_i = \{t_j : t_j =]j / \chi[, x_{ij} = 1,\ j = \overline{1,n}\};$

T_i is the number of years of product *i* life cycle.

To accommodate the change in prices over the life cycle the discount α_t is introduced.

$a_k = (1 + E_H)^{t_p - k}$, E_H - is a costs norm at different periods., t_p is a due year, $a_{t_i^{'}} = 0$ when $i \in \Phi$

$p_{i,prev}$ is the quantity of product *i* that was previously developed;

S_0 – funds for the preparation of all types of products production available due to the plannig period;

$b_t^{(h)}$ is the productive capacity's value for product *h* in *t* year;

$a_t^{(q)}$ is a parameter that is estimated by using the analytic function, algorithm or simulation model. $e_t^{(q)}$ is the $a_t^{(q)}$ parameter's claim in *t*-year.

$p_{ij} = 0,\ z_{ij} = 0$, if the product *i* can not fullfill the demand of the consumer *j*.

The various types of constraints can be used in the model and can be defined as analytical funtions, algorithmic or simulation models.

This dynamic problem belongs to the class of discrete optimization and contains boolean variables, algorithmic objective function, where various types of constraints such as analytical functions, algorithmic and simulation models can be used.

The search for optimal solutions constitutes the most important problem in the scope of discrete optimization. Improving the search efficiency is of considerable importance since exhaustive search is often impracticable.

The proposed direct search method consists of an intelligent iterative search that is based on upper bounds definition and avoids visiting those potential decision subsets, which are known not to contain an appropriate solution. The given method finds an exact optimal solution in reasonable time.

Description of the proposed direct search method

1. Define the profit upper bound when servicing all the consumers starting from *(j+1)* to *n*, $j = \overline{1,n}$.

The following equation defines only a profit upper bound for a single *j* consumer, $j = \overline{2,n}$.

$$\Pi_j^{\max} = \max_i \{[Ц_i(1) - v_i(\infty)]\, p_{ij}\}_{i=1}^{m}$$

Now, we need to find upper bounds that are required for computation of partial decisions vector, which consists of *j* coordinates, $j = \overline{1, n-1}$.

$$\Delta П_j = \sum_{k=j+1}^{n} П_k^{\max} \alpha_{]k/\chi[},$$

If $j = n,$ then $\Delta П_j = 0$.

Before we go to the next step, it is necessary to set values of the following variables $Д_0 = 0,\ V_0 := 0,\ W_0 := 0,\ Z_0 := 0$. If $x_{jk}^0 := 1$, then set $y_{j:} := k \quad \forall j = \overline{\chi(1-g)+1, 0}$, where $y_{j:}$ - is a coordinate of the current decision vector. This decision vector also includes prehistory decisions,

$$Y = (y_{\chi(1-g)+1}, y_{\chi(1-g)+2},\ \dots\ , y_0, y_1, \dots, y_n)^T$$

Set the initial record value for the objective function $П_{record} = -\infty$ and start servicing the first consumer request, so set *j*=1.

2. Try not to use any products for servicing the consumer *j*. This means setting the current decision vector's coordinate y_j and current product *i* as follow $y_j = i = 0$.

3. If $j \in U$, then set *i*:=*i*+1 and go to step 4. If $j \notin U$ and *i*=0, go to step 5.

4. Check fulfillment of the constraints.

4.1 If $p_{ij} = 0$, then set $y_j = 0,\ \ x_{ij} = 0$ and go to step 8.

4.2 Set $y_j = i,\ \ x_{ij} = 1$. Check fulfillment of the (2) constraint. If $j < n$, then check the constraints for the following year $]j/\chi[= t$. At the same time set $w_i\left(\sum_{j=1}^{n} p_{ij} x_{ij}\right) = w_i(p_i)$, if the product *i* has not yet been designed. If *j*=n , then check fulfillment of the (2) constraint for all years $t = \overline{1,T}$. If any one of the constraints is not met, set $y_j = 0,\ \ x_{ij} = 0$ and go to step 8.

4.3 Checking fulfillment of restrictions (3) in the year *t* by using the algorithm production capacity or through a simulation model. If any one of them is not met, set $y_j = 0,\ \ x_{ij} = 0$ and go to step 8.

4.4 Checking fulfillment of the algorithmic constraints (4) in year *t*. If any one of them is not met, set $y_j = 0,\ \ x_{ij} = 0$ and go to step 8.

5. Determine appropriateness of the decision.

5.1 Calculate the costs Z_j, V_j, W_j and the value of income $Д_j$.

5.2 Calculate $П_j = Д_j - W_j - V_j - Z_j$ and find values for the profits upper bound $\overline{П} = П_j + \Delta П_j$.

5.3 If $\overline{П} \le П_{record}$, this means that using the product *i* for the servicing the consumer *j* does not provide a better profit value than the record profit value, that has been determined before.

Then, set $y_j = 0,\ \ x_{ij} = 0$ and go to step 8.

6. If $j < n,$ then consider the next consumer request. Set *j:=j+1* and go to step 2.

7. Store the obtained profit record value $\Pi_{record} := \overline{\Pi}$ and its corresponding decision vector $R(j) = y_j\ \ \forall j = \overline{1,n}$

8. If i<m, then consider the next product i. Set *i:=i+1* and go to step 4.
9. If j>1, then consider the previous consumer request. Set *j:=j-1, i=*y_j and go to step 8.

When *j*=1, then if $\Pi_{record} > -\infty$ this indicates that solution has been found, otherwise the problem has no solution.

Proof of the valid upper bound definition
When calculating the upper bound while servicing the consumer request j, the maximum possible revenue value $Ц_i(1)p_{ij}$ and the lowest possible costs $v_i(\infty)p_{ij}$ are used. It can be argued that the mutual influence of servicing the consumers by various products can not give a profit value higher than the obtained upper bound. This is a true statement, because product price depends on its production volume, and the price of product produced in any number of units can not be higher than the price of product produced in single quantity $Ц_i(1)$. The production cost of product produced in any number of units can not be less than the production cost for product produced in a maximum possible quantity $v_i(\infty)$.

A computational investigation of the given method

A computational investigation of the method was carried out to show the method's efficiency.

Let's assume that we have to find an exact solution to a problem with 60x15 size. Using an exhaustive search to find an exact solution to this problem takes 1,17*10^{54} years, if the time for checking each possible solution is 10^{-9} sec. The results of the given method's computational investigation are shown below and prove the method's search efficiency. The computational investigation was done using a computational tool that will be further described in this paper. A desktop computer with the following hardware configuration was used: AMD Athlon 64 1.8 Ghz, 1GB RAM.

Table 1. The results of the computational investigation

Average solution search time, sec.	Problem size			Number of problems that have been solved
	n	m	nxm	
2	10	6	60	5
4	20	6	120	5
9	30	6	180	5
17	40	6	240	5
32	50	6	300	5
48	60	6	360	5
4	10	10	100	5
6	20	10	200	5
15	30	10	300	5
29	40	10	400	5
56	50	10	500	5
116	60	10	600	5

4	10	15	150	5
9	20	15	300	5
23	30	15	450	5
46	40	15	600	5
105	50	15	750	5
237	60	15	900	5

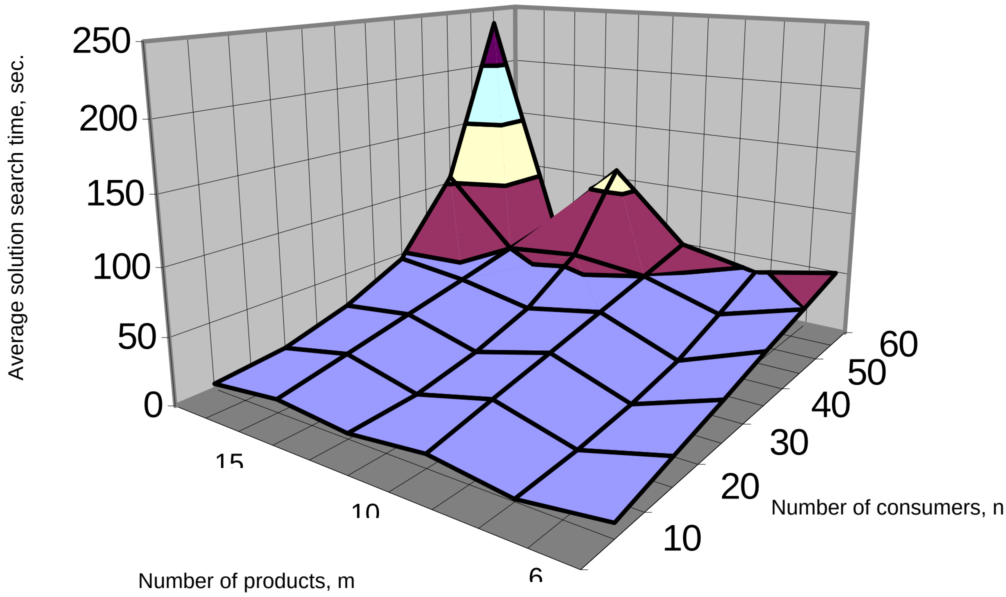

Figure 1. Problem size and its solution search time

Main stages in problem solving process and modular design application

Stage 1. This stage requires carrying out market research and calculating of forecasted demands for products that will meet the needs of the market.

Stage 2. Modules definition

A product can be assembled from the set of various predefined modules. For instance, in computer production the following components can be considered as modules: CPU, RAM, HDD, etc.

Each module has various features and specification figures such as performance, reliability index, demand parameter, economic figures, competitive figures, etc.

At this stage a set of possible modules are defined that will be further used for products' design.

Stage 3. Selection of product clusters

Selection of product clusters is based on forecasted demands, and products' features and specification figures. For example, in computer manufacturing products can be grouped into the following clusters: computers for scientific research (multiple CPU, a lot of RAM), servers (multiple CPU, a lot of RAM, high capacity HDD, RAID), play stations (high performance CPU, high capacity HDD, a lot of video memory and RAM), office computers (high hardware requirements are not needed).

Stage 4. Finding out features and specification figures of products that will meet the needs of selected product cluster.

Stage 5. Design various product items based on the predefined modules.

At this stage by combining different predefined modules we determine specific products that will meet the needs of product clusters.

Stage 6. Solving the multi-item capacitated lot-sizing problem in order to compute a maximum-profit production plan such that all customer orders are met in time.

Using a computational tool

Within scope of the research, a computational tool was developed. The tool was used for the computational investigation of the given method. As it was described above, the detailed market research and informatiom preparation are required for getting accurate input data. Let's assume that we have all the required input data. The computational tool has a number of forms for setting the input data prior to the problem's solution search. The main input parameters are described below. First, we set the matrix of forecasted demands that shows products' production quantity that is required to fulfill the demands of consumers.

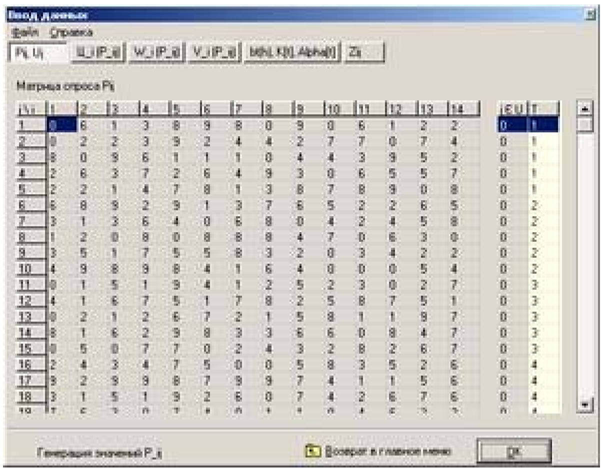

Figure 2. The matrix of forecasted demands

The next step is setting the matrixes of products price $Ц_i\left(\sum_{j=1}^{n} p_{ij}x_{ij}\right)$, the preproduction costs matrix $w_i\left(\sum_{j=1}^{n} p_{ij}x_{ij}\right)$, and the the production costs $v_i\left(\sum_{j=1}^{n} p_{ij}x_{ij}\right)$. All these matrixes are represented as functional dependecies and depend on the production volume.

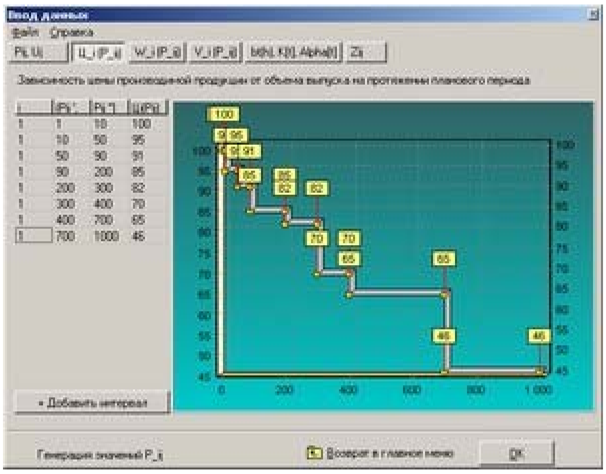

Figure 3. The products price matrix

When the process of inputing the data is accomplished, we can start the solution search process.

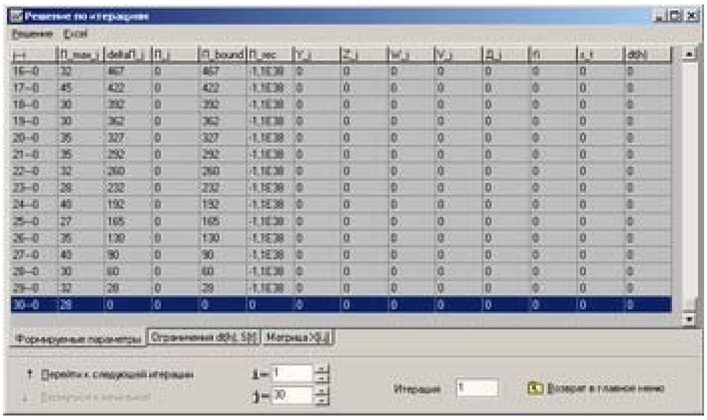

Figure 4. Solution search window

7. A practical application of the given approach

A company that is specialized in computer manufacturing is planning production of 4 basic types of products.

Type 1 - Servers (market value - $5,000.00)

Type 2 - Laptops (market value - $ 1,500.00)

Type 3 - Game stations (market value - $1,000.00)

Type 4 - Office computers (market value - $500.00)

Based on the marketing research data, the matrix of forecasted demands p_{ij} was determined. The production planning period is 3 years.

According to the marketing analysis, it is anticipated that the maximum number of customer orders per one year χ will be 100 units. Thus, the dimension of the matrix of forecasted demands p_{ij} (m x n) is 4 x 300.

There are $500,000.00 of existing funds S_0 prior to the planing period that can be used to cover the preproduction and any other further costs.

Additional capital investments in the first, second and third year will be $100,000.00.

The α_t discount value is 15%. The production planning period $T_i = 3$ years.

The functions that represents products' prices $Ц_i\left(\sum_{j=1}^{n} p_{ij}x_{ij}\right)$, the preproduction costs $w_i\left(\sum_{j=1}^{n} p_{ij}x_{ij}\right)$, and the the production costs $v_i\left(\sum_{j=1}^{n} p_{ij}x_{ij}\right)$ are defined in the form of nonlinear dependence.

Table 1Functional dependencies of the product price, preproduction costs, production costs and the production volume of the product "Servers"

Pi interval		Vi	Цi	Wi
1	4	3610	5198,4	24000
5	10	3590	5169,6	24200
11	20	3550	5112	24400
21	30	3500	5040	40000
31	40	3470	4996,8	40000
41	50	3450	4968	52000

Table 2 Functional dependencies of the product price, preproduction costs, production costs and the production volume of the product "Laptops"

Pi interval		Vi	Цi	Wi
1	4	1100	1584	24000
5	10	1080	1555,2	24200
11	20	1060	1526,4	24400
21	30	1042	1500,48	40000
31	40	1010	1454,4	40000
41	50	990	1425,6	52000

Table 3 Functional dependencies of the product price, preproduction costs, production costs and the production volume of the product "Game stations"

Pi interval		Vi	Цi	Wi
1	4	735	1058,4	24000
5	10	725	1044	24200
11	20	715	1029,6	24400
21	30	695	1000,8	40000
31	40	670	964,8	40000
41	50	640	921,6	52000

Table 4 Functional of between the product price, preproduction costs, production costs and the production volume of the product "Office computers"

Pi interval		Vi	Цi	Wi
1	4	397	571,68	24000
5	10	380	547,2	24200
11	20	365	525,6	24400
21	30	350	504	40000
31	40	320	460,8	40000
41	50	297	427,68	52000

Table 5 Productive capacity constraints

t \ i	1 (Servers)	2 (Laptops)	3 (Game st.)	4 (Office comp.)
1	150	300	400	600
2	150	300	400	600
3	150	300	400	600

Zij, the costs of transportation, adjustments, and possible setup costs, is $ 20.00 per unit of the product.

The company also has contractual obligations with some customers for computers supply $j \in U$, where U is a contractual obligations matrix.

As a result, the maximum-profit production plan was found. The maximum possible profit Π_{record} is $ 562, 042.

The obtained production plan is shown below.

According to solution of the problem it follows that during the first year the company needs to fulfill 28 customer orders by supplying 23 units of type-1, 250 units of type -2, 60 units of type-3 and 200 units of type-4.

During the second year it is going to fulfill 30 customer orders and needs to produce 42 units of type-1, 245 units of type-2, 50 units of type-3 and 110 units of type-4.

In the third year it is going to fulfill 19 customer orders and it is necessary to produce 44 units of type-1, 150 units of type-2, 30 units of type-3 and 465 units of type-4.

Conclusion

This research paper reviews the problem of dynamic multi-item capacitated lot-sizing problem from the point of commercial enterprises. The model and the direct search method were proposed. The given method finds an exact solution in reasonable time. The main stages in the problem solving process and modular design application were described. A computational tool was developed and used for the computational investigation of the given method. The results of the research show method's search efficiency and its practical application value for commercial enterprises as it is able to determine a maximum-profit production plan.

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