ALGEBRA LOGIC APPROACH TO PERSON'S THINKING MECHANISMS FORMALIZATION

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Abstract: It is known, that person's thinking is inaccessible to studying by direct physical and psychological methods. In this case it is necessary to have indirect ones. Computers do not understand the psychological description and formalization of thinking mechanisms. Algebra logic analysis of natural language and person's thinking plays an important role for development of logic mathematics and its applications in artificial intelligence. Only axiomatic method works in this situation. On the basis of axioms' system we can propose an approach that helps to investigate the structure and properties of objects. The main problem of formal studying of a natural language is shortage of the mathematical apparatus. The axiomatic description of logic mathematics' objects requires preliminary realization of constructive logical tools, which subsequently become a subject of the axiomatic analysis. The paper is devoted the algebra of ideas to axiomatic construction. The carrier of this algebra is naturally interpreted as the set of intelligence ideas (thoughts, concepts and, in general, any subjective conditions of the person). There are devised some methods for application of proposed formal apparatus. Simultaneously with algebra of ideas formal introducing there is considered its intentional interpretation.

Keywords: a predicate, algebra of ideas, artificial intelligence, Cartesian set, algebra of predicates, algebra of single k-dimentional first order predicates.

ACM Classification Keywords: I.2 Artificial intelligence - Natural Language Processing

Introduction

One of artificial intelligence lacks, which is much limiting sphere of its practical use, is the inability of the machine to understand human speech and, as a consequence, impossibility of semantic processing of the natural language texts. For studying of human thinking, in particular, of natural language semantics mechanisms are successfully used comparative method and logic apparatus of predicates and predicate operations. [Bondarenko, 2000] For development of logic mathematics and its appendices in artificial intelligence the special role plays algebra logic analysis of natural language. In this case we can apply axiomatic method. On the basis of axioms' system we can propose an approach that helps to investigate the structure and properties of objects. The axiomatic description of logic mathematics' objects requires preliminary realization of constructive logical tools, which subsequently become a subject of the axiomatic analysis. Yet there are no many abstract concepts for description of natural intellectual processes. [Shabanov-Kushnarenko, 2005]

In logic mathematics the central role is played by relations. The relation are formally described with the help of predicates. [Ivanilov,2007] In language of algebra of predicates it is possible to describe any information process but the algebra of predicates is constructed structurally. It is enough, if the internal structure of information process is known for us, but it is not enough, if we have only results of this process, as in natural language. What occurs inside, what algorithms are working – it is not known. That is why it is necessary to set algebra of predicates axiomatically. Then these properties we can observe in real human speech and behavior and to make conclusions about structure of these information processes.

The algebra of predicates has appeared as a result of attempts of the formal description of natural language - modeling of declinations, conjugations, words' formation. Now, there is more complex task - formalization of understanding and semantics of language. In this case it is necessary to describe concept of a predicate axiomatically because relations, but not functions, lays in the basis of thinking. The task about the formal description of a predicate is naturally divides into two tasks. The first is a consideration of a single predicate from variable x, and the second is expansion of this integrated variable x in a set of variables, i.e. studying of a many-place predicate or structure of the Cartesian product.

The paper is devoted to construction of the methods of formal description of natural language structure with the help of algebra of ideas - mathematical apparatus, which is constructed axiomatically as algebra logic analogue of natural language. The urgency of this area is defined by perspectives of applying of the received methods for developing systems of dialogue with the computer in natural language. In this work the properties of single predicates are considered, as it is enough for modeling rather wide area of natural language. Except studying of single predicates, there are some adjacent questions, such as predicate of equality and models, i.e. circle of tasks closely connected with axiomatic of a single predicate.

A model of ideas' equality. Formal representation of ideas

We shall use algebra of single k-dimentional predicates of the first order in a role of the algebra of ideas prototype. It appears, that exactly the algebra of single k-dimentional predicates of the first order brings to the most general algebra of ideas definition that is necessary to us. Abstract analogues of the more general algebras of final predicates (many-placed and the any order) turn out simply by detailed elaboration of initial algebra of ideas.

single k-dimentional predicates of the first order are entered as follows. Let $A_k = \{a_1, a_2, ..., a_k\}$ is the set, that consists of k letters $a_1, a_2, ..., a_k$. All letters are numbered, everyone has the serial number. The variable x is set on A_k and it named alphabetic. We enter the set $\Sigma = \{0, 1\}$ that consists of logic constants 0 and 1, named accordingly zero and unity. The variable x is set on Σ and it named logic. Each function $y = P(x)$ that display set A_k in set Σ we named as single k-dimentional predicate of the first order. Let's speak, that predicate P is set on set A_k. Set of all single k- dimentional predicates of the first order we designate by a symbol M_k. Let $N_0(k)$ is a number of all predicates included in set M_k. It is equal $N_0(k) = 2^k$.

algebra of ideas Construction we shall begin with introducing of its carrier - set of all ideas. We shall designate by a symbol S_k the set consisting of 2^k various elements $s_0, s_1..., s_{2^k-1}$. we Accept the set S_k in a role of the algebra of ideas carrier with dimension k. Elements of set S_k we name ideas of dimension k. Single k-dimentional predicates of the first order serve for us as prototypes of elements of set S_k. The number of elements 2^k of set S_k is chosen so that it coincided with number of all single *k*-dimentional predicates of the first order. Set S_k we shall name k-dimensional space of ideas. The question on concrete value of number k is left open. While we shall consider, that in a role k any natural number *k*=1, 2, … can be chosen. Let's notice, that at any value k the set S_k is not empty. In some tasks we need not all the set S_k but only some part N of it. The number of elements in set N can be any, but it should be less, than 2^k. Set N we shall name incomplete set of ideas, and set S_k - full.

Let's enter bijection $\Phi: S_k \to M_k$, establishing univocity between all ideas of dimension k and all k-dimentional predicates that set on set A_k. It always can be made, because sets S_k and M_k contain identical number of elements. Predicate $P = \Phi(x)$ we shall name a predicate corresponding to idea x, and idea $x = \Phi^{-1}(P)$ - the idea corresponding to predicate P. There are two examples of bijection Φ'and Φ" in tables 1 and 2. Bijection Φ': $S'_k \to M_k$ is determined on three-dimensional space of ideas $S'_3 = \{s'_0, s'_1, ..., s'_7\}$, bijection Φ'': $S''_k \to M_k$ is determined on space of ideas $S''_3 = \{s''_0, s''_1, ..., s''_7\}$ with the same dimentional. The symbol x ' designates variable that sets on set S'_3 and symbol x " - a variable that sets on set S''_3. Sets S'_3 also S''_3 can be considered as different systems of designations for the same three-dimensional ideas.

Table 1 Variables that set on set S'_3

x'	s'_0	s'_1	s'_2	s'_3	s'_4	s'_5	s'_6	s'_7
$\Phi'(x')$	P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7

Table 2 Variables that set on set S''_3.

x"	s''_0	s''_1	s''_2	s''_3	s''_4	s''_5	s''_6	s''_7
$\Phi''(x'')$	P_4	P_5	P_7	P_1	P_6	P_0	P_3	P_2

Elements of set S_k we shall psychologically interpret as ideas of the examinee. Predicate P (the question is about single k-dimentional predicates of the first order) that accept for all letters $x \in A_k$ zero value $P(x) = 0$, we shall name identically false. Predicate P that accept for all letters $x \in A_k$ individual value $P(x) = 1$, we shall name identically true. We designate these predicates accordingly symbols 0 and 1. The predicate 0 has number 0, a predicate 1 - number $2^k - 1$.

The idea that corresponds to identically false predicate 0 we shall lie, and designate it by the same symbol 0. The idea that corresponds to identically true predicate 1, we shall name true and designate it by symbol 1. Thus, $\Phi^{-1}(0) = 0, \Phi^{-1}(1) = 1.$ Operation of the bijection Φ reference is designate by symbol $^{-1}$. In a role of function Φ^{-1} arguments they mean predicates, that are the elements of set M_k, and in a role of function Φ^{-1} values they mean ideas, that are the elements of set S_k. This circumstance, however, will not result in misunderstanding because the true sense of signs 0 and 1 is easily determined on a context. For example, we shall find ideas 0 and 1 in sets S'_3 and S''_3 with help of tables 1 and 2. In a role of the predicate 0 acts predicate P_0, in a role of the predicate 1 - predicate P_7 in both tables. We find $\Phi'^{-1}(P_0) = s'_0$, $\Phi'^{-1}(P_7) = s'_7$. from table 1. Thus, for set S'_3 we have $0 = s'_0$, $1 = s'_7$. We find $\Phi''^{-1}(P_0) = s''_5$, $\Phi'^{-1}(P_7) = s'_2$ from table 2. Thus, for set S''_3 we have $0 = s''_5$, $1 = s''_2$.

Statement that express lie, we shall name the contradiction. The statement that express true, we shall name a tautology.

A predicate of ideas' equality

Let's consider a predicate of equality $D_k(P, Q)$ of predicates P and Q, which are set on the Cartesian square of set M_k of all single k-dimentional first order predicates.

It defines by equality:

$$D_k(P, Q) = \forall x(P(x) \sim Q(x)), \tag{1}$$

that fair for anyone $P, Q \in M_k$. The predicate D_k puts in conformity to equal predicates P and Q a logic constant 1, unequal - 0. The equation $D_k(P, Q) = 1$ sets the relation of equality $P = Q$ of predicates

$P, Q \in M_k$. The equality relation of predicates can be considered as the diagonal relation set on the Cartesian square of set M_k, i.e. as set of all pairs a kind (P, P) where $P \in M_k$. In our example the set $\{(P_0, P_0), (P_1, P_1), ..., (P_7, P_7)\}$ serves as equality relation. the Equation $\mathsf{D}_k(P, Q) = 0$ sets the inequality relation $P \neq Q$ predicates P and Q. The inequality relation of predicates can be considered as the antidiagonal relation that set on the Cartesian set M_k.

Let's introduce a predicate of equality of ideas D_k on set $S_k \times S_k$, defining it for anyone $x, y \in S_k$ as follows:

$$D_k(x, y) = \mathsf{D}_k(\Phi(x), \Phi(y)). \tag{2}$$

Here Φ is bijection that display set S_k on set M_k. The predicate $D_k(x, y)$ predicate $D_k(x, y)$ displays set $S_k \times S_k$ on set Σ. Being sent from definition (2) and using equality and inequality relations of predicates, we can present a predicate D_k as

$$D_k(x, y) = \begin{cases} 0, \text{if } \Phi(x) \neq \Phi(y), \\ 1, \text{if } \Phi(x) = \Phi(y). \end{cases} \tag{3}$$

Let's consider two models $\langle S_k, D_k \rangle$ and $\langle M_k, \mathsf{D}_k \rangle$. First of them represents set S_k together with the predicate D_k set on its Cartesian square, another - set M_k together with the predicate D_k set on its Cartesian square. Equality (1) means, that models $\langle S_k, D_k \rangle$ and $\langle M_k, \mathsf{D}_k \rangle$ are isomorphic each other. The relation of isomorphism of models is equivalence.

We shall make some specifications of the introduced terminology. Ideas we shall name, in the first place, mathematical objects - elements of set S_k, at the second place, psychological objects - any subjective conditions of the person. In the second meaning the term idea we shall use only at the expanded statement of tasks of the intelligence theory. We shall name psychological objects by ideas - all those subjective conditions of the person which can be expressed in the form of statements.

The signals showed to the examinee during carrying out of experiences, we shall name physical stimulus. We shall speak, that physical stimulus serve as prototypes of ideas, and ideas are images of physical stimulus. At narrow problem definition in a role of physical stimulus will act statements, and in a role of their images will act only ideas. At expanded problem definition stimulus can be any physical objects.

Properties of a equality predicate of ideas

Let's consider properties of a predicate D_k. It submits to laws of reflexivity, substitution, symmetry and transitivity. In formal record these laws look like the following logic equations:

$$\forall x\, D_k(x, x) = 1, \tag{4}$$

$$\forall x \forall y\, (D_k(x, y) \supset D_k(y, x)) = 1, \tag{5}$$

$$\forall x \forall y \forall z\, (D_k(x, y) \wedge D_k(y, z) \supset D_k(x, z)) = 1, \tag{6}$$

$$\forall R_k \forall x \forall y\, (R_k(x) \wedge D_k(x, y) \supset R_k(y)) = 1, \tag{7}$$

Here, variables x, y, z are set on set of all ideas S_k, the variable R_k is define on set of all predicates which are determined on set S_k. The variable predicate connected by the logic equations (4) - (7) designates by symbol D_k.

We have defined a equality predicate of ideas D_k and have deduced its four properties, being sent from a equality predicate of predicates (1) and using expression (2). However, it would be desirable to construct the approach of equality of ideas on the bases, not dependent on concept of a final predicate which in our statement carries out only auxiliary role of the prototype of concept of idea.

As it is proved in the statement resulted below, it can be made, basing on properties (4) - (7) of equality predicates of ideas as on axioms. Value of the statement will be, that it gives axiomatic definition of a predicate of equality of ideas.

Statement 1.

To present in form (1) predicate D_k, that defines on set $S_k \times S_k$, it is necessary and enough that it satisfied to conditions of reflexivity, symmetry, transitivity and substitution.

That is why any two models that isomorphic the third are isomorphic each other. We shall take models $\langle S'_k ,\ D'_k \rangle$ and $\langle S''_k , D''_k \rangle$. Both of them are isomorphic to model $\langle M_k ,\ \mathsf{D}_k \rangle$, so they are isomorphic to each other.

From here follows the existence of bijection $\Omega :\ S'_k \to S''_k$, for which at anyone $x,\ y \in S'_k$ takes place the equality:

$$D'_k (x,\ y) = D''_k (\Omega(x),\ \Omega(y)). \tag{8}$$

Expression (8) means, that in abstract sense predicates of ideas equality, and, consequently, relations of ideas equality, that appears in any algebras of ideas of the same dimension, are indistinguishable from each other. Insignificant distinction from the mathematical point of view consists only in a concrete way of a designation of elements of set S'_k and S''_k of carriers of these algebras. If we replace names of set S'_k elements with names of set S''_k elements by bijection Ω the predicate of ideas equality D'_k, which is set on set $S'_k \times S''_k$ will turn in a predicate of ideas equality D''_k, which is set on set $S''_k \times S''_k$.

The equality predicate $D_k(x,\ y)$ of ideas x and y is practically realized by the examinee in a series of experiences. Every experience consists of researcher suggestion to examinee of two ideas $x = a$ and $y = b$ which are showed in the certain order so that examinee always knows what is the first of them and what is the second. He needs to compare the ideas showed to him and to establish, they are equal or not. In case of full concurrence of ideas *a* and *b* the examinee reacts the answer 1 if they are differ in something the answer will be 0. Experience shows, that the examinee recognizes two ideas equal in all those and only those cases when statements that express these ideas are logically equivalent.

When we define the algebra of ideas formally, we have introduced the set of all ideas S_k and only after that have set on it an equality predicate $D_k(x,\ y)$ for any ideas $x,\ y \in S_k$. At substantial introduction of algebra of ideas (i.e. such algebra of ideas at which a role of ideas play ideas of the person) it is necessary to make on the contrary: first to introduce an equality predicate of ideas, and then the set of all ideas with help of this equality predicate.

The researcher has no direct access to ideas of the examinee. Therefore he is compelled to find set of ideas of the examinee, basing exclusively on supervision results of examinee behavior. The researcher can act as follows. He shows to the examinee various pairs physical signals which from his point of view can carry out a role of ideas names, and suggests examinee to establish, are equal or not ideas that corresponding to these signals.

Thus the researcher, first of all, should find out, is examinee capable to react on those or other pairs of signals. If it appears, that examinee always reacts by quite certain answer on some pair of entrance signals, the researcher, should establish, will be a reaction of examinee on this pair of signals unequivocal or not.

With this purpose the researcher in a random way between other pairs of signals, repeatedly shows the same pair of signals that is interesting for him. If the examinee reacts once to this pair of signals the answer 0, and other time - the answer 1 the signals of such pair should not be included in structure of set S_k as names of ideas. So, using a predicate of equality as the tool, the researcher forms the set of all ideas for the given examinee. It is necessary to specify, that actually the researcher collects in set S_k not ideas of the examinee, but names of these ideas. If for any idea have been used several different names, the researcher select only one of them. If the researcher puts before itself any private tasks he can be limited to revealing not all ideas of the examinee but only some part of then that is interesting for him, for example, ideas of mathematical character.

Conclusion

On the basis of algebra single k-dimntional predicates of the first order is offered the algebra of ideas that intended for formalization of subjective conditions of the person.

The algebra of ideas structure is developed: the carrier of algebra and its axiomatics.

The equality predicate of ideas is introduced as the tool for experimental studying ideas of the person, the axiomatics of this predicate is determined.

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