MINIMUM MEAN SQUARE ERROR APPROACH IN IMAGES PROCESSING FOR FULL-FIELD DISPLACEMENTS AND DEFORMATION MEASUREMENTS

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Abstract: A vision system is applied to full-field displacements and deformation measurements in solid mechanics. A speckle like pattern is preliminary formed on the surface under investigation. To determine displacements field of one speckle image with respect to a reference speckle image, sub-images, referred to Zones Of Interest (ZOI) are considered. The field is obtained by matching a ZOI in the reference image with the respective ZOI in the moved image. Two image processing techniques are used for implementing the matching procedure: – cross correlation function and minimum mean square error (MMSE) of the ZOI intensity distribution. The two algorithms are compared and the influence of the ZOI size on the accuracy of measurements is studied.

Keywords: Full-field displacement measurement, Image correlation, Minimum mean square error of intensity distribution.

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Introduction

The development of reliable, affordable, easy to use full-field measurement techniques is crucial when one wants to characterize the mechanical effects on solids and structures. Materials which are homogeneous at the scale of observation but subject to complex load histories or heterogeneous lead to kinematics fields that need multi-scale analysis to relate the microstructure to the mechanical response. The full-field measurements allow a synergism of two areas that for a long time lived in separate environments – experiments and simulations, to be established.

The fast development of CCD or CMOS cameras that can be used at various scales enables for rapid dissemination of contactless techniques for full-field measurements. Usually these measuring techniques are based on image processing algorithms using correlation principles that were initially introduced in fluid mechanics since the development of Particle Image Velocimetry (PIV) [1, 2] in the 1970's. It is also adapted in solid mechanics since early 1980's [3]. The texture heterogeneities at a surface (or in a volume) of observation are utilized for determination of displacements fields using pattern matching algorithms. With the generalization of very powerful and affordable computers, these techniques start to be commonly used under various conditions [4].

A speckle like pattern is preliminary formed on the surface under investigation. To determine displacements field of one speckle image with respect to a reference speckle image, sub-images, referred to Zones Of Interest (ZOI) are considered. The field is obtained by matching a ZOI in the reference image with the respective ZOI in the moved image.

In cases when the mean values of visibility (contrast) of regions possessing a ZOI dimension is significantly different, the usage of cross- correlation algorithms leads to increasing the probability for a wrong identification the corresponding ZOI of the second image. Such a situation may be a consequence of a non uniform illumination, as well as of non equal reflective properties of observant surface. It is appropriately in these cases an algorithm, based upon minimum mean square error (MMSE) calculation to be used.

Algorithm

The deformation field of the investigated surface can be obtained when two surface images are compared. The first image is stored in computer before the object deformation. It is considered as reference image. A second image is registered after deformation of the observed surface. For every pixel from the reference image an area surrounded that pixel is called zone of interest (ZOI). To determine the vector field of deformation one has to match the corresponding ZOI-s in the two images. If the function of the deformation is a smooth function it is possible to skip calculations in a reasonable distance. A vector field of deformation, derived by the above described way is shown in fig. 1

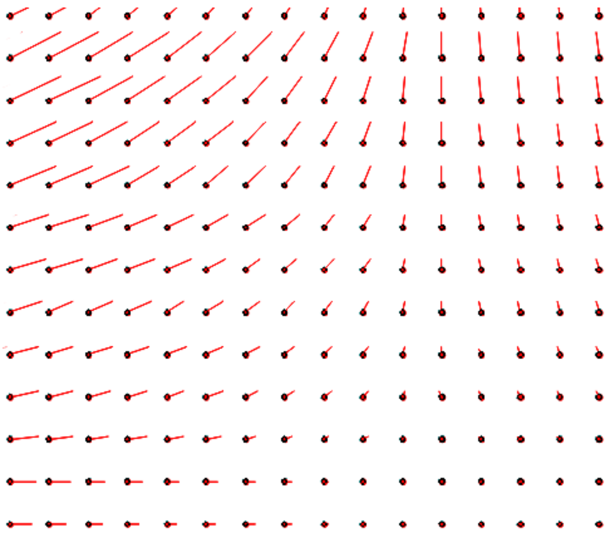
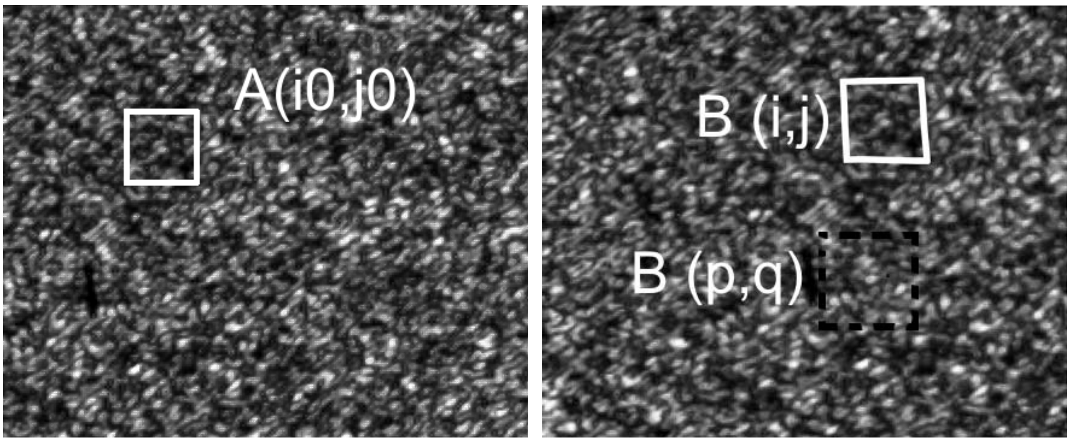


Fig. 1 A vector field of deformation

Let us consider a ZOI $A(i_0,j_0)$ in the reference image (the square zone in Fig. 2-a).



a b

Fig. 2. (a, b) Two consecutive images of the investigated surfaces, registered in the process of its deformation, a- reference image, b- deformed image

One can solve the correlation problem for finding the location displaced ZOI A(i0,j0) to a ZOI B(i,j) position (Fig. 2b – the deformed square zone), accepting an assumption, that the whole surface of interest is illuminated and reflects the light so that no ZOI-s of the image with different mean intensity value. Under this condition ZOI B(i,j) can be searched for in a near vicinity of ZOI A(i0,j0), having a radius equal to the maximum expected reallocation. Calculating the 2D correlation function the ZOI B(i,j) is found with a big degree of a probability at this location at which maximum value of correlation (a correlation peak) is obtained during the above procedure performance. There exists however situation in which the assumption of uniform, mean image intensity is not satisfied. A wrong correlation maximum can be obtained at position of ZOI B(p,q) (the circular zone in Fig. 2b), due to the fact that the mean intensity of the second image at this position exceeds the mean intensity value of the entire image. As a result the degree of probability of true correlation peak location decreases.

The algorithm of ZOI matching using MMSE calculation instead of correlation function is an acceptable alternative for overcoming this difficulty. By usage of the algorithm a “correlation” peak is generated only when the ZOI A(i_0,j_0) and ZOI B(i,j) values are identical and as an amplitude as positions of pixels corresponding to the zones.

Theory

According to correlation theory a narrow autocorrelation peak can be obtained by processing of a large ZOI. This is a guarantee of precise localization of the peak. At the same time it is well known that the number of calculations increases by power 2 of the ZOI size. Looking for a numerical method, ensuring good enough accuracy in cases when small ZOI size is processed we have found that application of the standard correlation method leads to unsatisfied results in case of small ZOI size having low contrast of the intensity variations. The correlation formula (1) presents a linear function of the amplitude of discrete input signals.

$$C_{(i0,j0,i,j)} = \sum_m \sum_n A_{(m+i0,n+j0)} . B^*_{(m+i,n+j)} \quad (1)$$

Where *A* is a ZOI before deformation and *B* is the image after deformation, *C* is the signal cross- correlation function, *i0* and *j0* are indexes of the reference ZOI A images when a Cartesian coordinate system is used, *i* and *j* are the corresponding indexes in the same coordinate system of the corresponding ZOI in B, m and n are indexes of the digital integration. B* is a conjugation value of *B*. The multipliers which are independent of the processed information are ignored.

$$C_{(i0,j0,p,q)} = \sum_m \sum_n A_{(m+i0,n+j0)} . B^*_{(m+p,n+q)} \quad (2)$$

If B is presented as multiplication (3) of a normalized function N and the mean contrast for this ZOI the two correlation functions can be compared as function of the contrast (4).

$$B_{(m+i,n+j)} = k_{(i,j)} . N_{(m+i,n+j)} \quad (3)$$

$$k(i,j) C^N_{(i0,j0,i,j)} < k(p,q) C^N_{(i0,j0,p,q)} \quad (4)$$

One can see from equation (4) that it is possible such a value of the quotient of the image contrasts K(i,j)/K(p,q) to be found that satisfy the inequality. This means that the correlation function value C(i0,j0,p,q) of ZOI A and B will have its maximum at an other ZOI B(p,q) with coordinates p and q that is not corresponding to the “true” solution C(i0,j0,i,j) in Fig. 2b.

In this example we have found ZOI B(p,q), similar to ZOI B(i,j). The wrong zone has a larger K (higher contrast of 2D intensity distribution) in comparison to the ZOI B(i,j). Since we are searching for the highest value of the correlation function, a “wrong” location of ZOI B with coordinates p and q could be determined. The utilization of MMSE approach leads to comparison of the signal according to both: its shape and its amplitude (equations (5) and (6)).

$$E_{(i,j)} = \sum_m \sum_n [A_{(m,n)} - B_{(m+i,n+j)}]^2 = \sum_m \sum_n e^2_{(m+i,n+j)} \quad (5)$$

The characteristic of the function of MMSE *E* can be seen easily If the contrast is presented as *1+k* that is a increasing function too.

$$E_{(i,j)} = \sum_m \sum_n [A_{(m,n)} - (1+k) N_{(m+i,n+j)}]^2 = \sum_m \sum_n ke^2_{(m+i,n+j)} = kE_{N(i,j)} \quad (6)$$

where *e* is the error between reference ZOI A and target ZOI B.

To obtain a function similar to the cross correlation function operations normalization and inversion are used.

$$C = 1 - \frac{E}{Max(E)} \qquad (7)$$

or

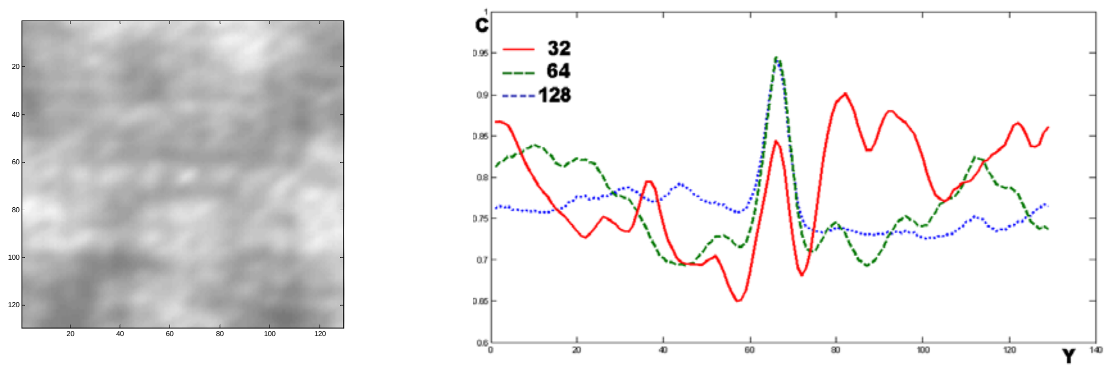
$$C = 1 - \frac{kE_N}{Max(Ek)} = 1 - k\frac{E_N}{Max(Ek)} \qquad (8)$$

It can be seen from equation (2) that at the correlation algorithm, if one of the input signals increases the correlation value increases as a result. When MMSE algorithm is implemented the increasing one of the input signals results to increasing of the error(6) and decreasing of the correlation peak value (8).

Experimental results

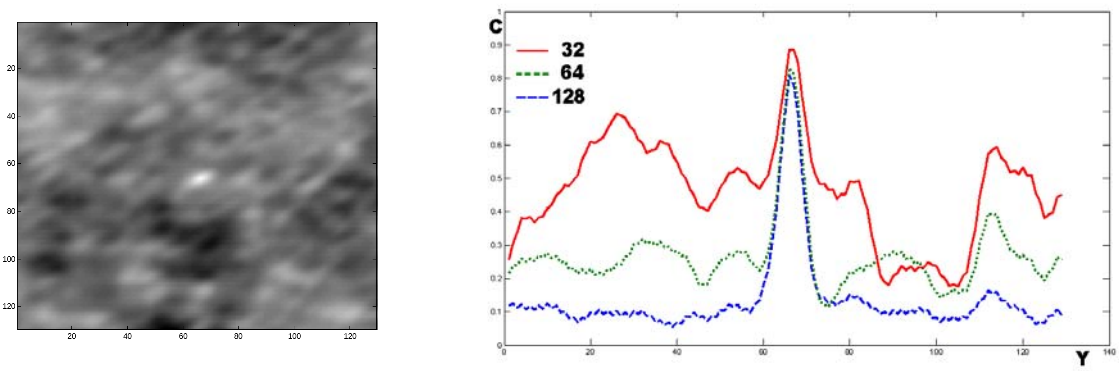
The above described algorithms for ZOI matching were optimized according to its parameters and were applied to the image shown in Fig. 2. The correlation maximum is calculated using the cross-correlation function for a square region of search for matching region 128x128 pixels and with ZOI size 32x32 pixels. Three ZOI size - 128, 64 and 32 pixels, are shown and results obtained are depicted in Fig 3-b. It is seen that the decreasing of ZOI size leads to increasing of the value of the correlation function at wrong position. At the ZOI size 32 pixels, the ”true” correlation peak value is smaller than the value of the peak obtained in the wrong location.

MMSE method is used for carrying out the similar calculations and the results are presented in Fig 4.



a b

Fig. 3. a - Correlation function calculated in a 2D region, b - a distribution of correlation values along to the mean vertical cross section of the region at 128, 64 and 32 pixel size of ZOI



a b

Fig. 4. a – Mean Square Error for a 2D region, b - a distribution of correlation values along to the mean vertical cross section of the region at 128, 64 and 32 pixel size of ZOI

Computer programs for realizing the correlation and MMSE algorithms are developed in C^{++} environment. The computing of cross-correlation function is carried out by a standard optimized algorithm, and of MMSE – by a gradient algorithm. The processing time of performing the MMSE calculations is about 50% longer than the time of correlation calculations.

Conclusion

The solution of ZOI correspondence problem by a cross-correlation approach is a fast and accurate way for determining of deformation vector fields in mechanics of solids. In case when relatively big deformations of second image , related to the reference image are observed, a reduction of the images area or decreasing of ZOI size have to be introduced for obtaining acceptable solution. The usage of MMSE approach leads to increasing the reliability of ZOI searching, which is paid by an increasing computing time. A combination of the two algorithms that would unit their advantages can be realized to reach optimal full-field measurements. Routine calculations my be carried out by the correlation algorithm and a switch to MMSE algorithm can be programmed for the image regions in which decreasing of correlation algorithm reliability is expected or estimated.

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